

BOOK REVIEWS

Markov processes and potential theory, by R. M. Blumenthal and R. K. Gettoor. Monographs in Pure and Applied Mathematics, Academic Press, New York, 1968. x+313 pp. \$15.00.

This is a very important book. It will certainly be, in the years to come, the basic text for teaching Markov processes, and the basic reference on this subject.

The theory of time continuous Markov processes consists nowadays of three main fields of activity. First we have much work done on time continuous Markov chains, that is on processes with a very simple discrete state space, under weak hypotheses allowing very discontinuous sample functions. Quite to the opposite, we find the theory of diffusion processes, concerned with nice sample continuous processes on manifolds. Blumenthal and Gettoor's book is devoted to the "middle" field, that of all "reasonable" sample right continuous Markov processes on locally compact spaces, along the lines of Hunt's fundamental papers,¹ that is, emphasizing the interplay with ideas and methods borrowed from potential theory. It is quite remarkable that, while so many people were doing research under Hunt's influence, this book is the first one to cover entirely the subject matter of Hunt's papers. Special thanks are due to the authors for having made accessible to all mathematicians the most cryptic parts of MP2 and MP3.¹

Let us first describe the contents of the book, starting with the chapters concerned with Hunt's work. Chapter I contains the background of Markov processes: Markov property, transition function, strong Markov property, standard processes and Hunt processes, measurability of hitting times, and the basic fact that processes with a Feller transition semigroup are Hunt processes. Chapter II contains the core of MP1: excessive functions and their behavior on sample paths, exceptional sets, fine topology. However, the main balayage theorem is excluded from this chapter and given in Chapter III with the "relative theory of potentials": subordinate semigroups and resolvents, subprocesses, special sets, multiplicative functionals. That is, Chapter III contains a much modernized version of MP2. Finally, the contents of MP3 are given in Chapter VI: dual processes,

¹ *Markov processes and potentials*. I, II, III, Illinois J. Math. 1(1957), 44-93; 316-396; 2 (1958), 151-213. We refer to these papers below as MP1, MP2, MP3.