ON THE CONSTRUCTION OF THE EILENBERG-MOORE SPECTRAL SEQUENCE

BY LARRY SMITH[1]

Communicated by Frank Peterson, January 13, 1969

Suppose that

$$\mathfrak{F} \qquad \begin{array}{ccc} E & \longrightarrow & E_0 \\ \downarrow & & \downarrow \\ B & \longrightarrow & B_0 \end{array}$$

is a fibre square. Under suitable regularity conditions Eilenberg and Moore have introduced [6] (see also [16], [17]) a spectral sequence $\{E_r(\mathfrak{F}), d_r(\mathfrak{F})\}$ with

$$E_r(\mathfrak{F}) \Rightarrow H^*(E; k),$$
$$E_2(\mathfrak{F}) = \operatorname{Tor}_{H^*(B_0;k)}(H^*(B;k), H^*(E_0; k)),$$

where $H^*(\ ;\ k)$ denotes cohomology with coefficients in the field k. This spectral sequence has proved to be a powerful tool for dealing with numerous problems in algebraic topology [3], [16], [17]. The constructions of [6] (also [16]), are entirely algebraic in nature and have left open several rather important points, among them the relation between the spectral sequence and the Steenrod algebra when $k = Z_p$, p a prime. (That such a relation should exist is strongly suggested by [16, §4] and [9], and has provided a large portion of the motivation for the present work.) We will remedy this shortcoming by providing a geometric construction of the spectral sequence, that will also clarify the status of the spectral sequence for generalized cohomology theories.

We will adopt the point of view, first employed by Hodgkin in [7] that the Eilenberg-Moore spectral sequence should be the Künneth spectral sequence for a suitable cohomology theory on a suitable category. This point of view contains a good deal more than the germ of the construction. For there already exists a method, geometric in nature, for dealing with Künneth type problems for general cohomology theories [2]. This method of Atiyah has recently come under rescrutiny and has been extended to more general situations [1; I], [5; §8], [7]. This suggests that we try to adopt Atiyah's method to the case at hand.

[1] During the course of this work the author was an Air Force Office of Scientific Research Postdoctoral Fellow.