

# ANALYTIC MAPS BETWEEN TORI<sup>1</sup>

BY HEINZ G. HELFENSTEIN

Communicated by F. W. Gehring, February 20, 1969

We obtain the conditions under which nonconstant complex analytic maps between two tori exist, as well as the set of all such maps. The arguments carry over to analytic maps between higher-dimensional tori, as well as other categories of maps; e.g. affine maps between flat manifolds. Since the precise statements of the results become complicated, even under simple circumstances, we restrict ourselves to the case of 2-dimensional tori. The main point of interest lies in the existence of mutually disjoint dense subsets of the manifold of all tori, which yield different sets of analytic homotopy classes of maps. Details will appear elsewhere.

Let  $GL^+(2, Q)$  denote the group of all  $2 \times 2$  matrices with real rational entries and positive determinant;  $SL(2, Z)$  the subgroup of all  $2 \times 2$  matrices with real integer entries and determinant  $+1$ . Then the groups

$$G = GL^+(2, Q) / \{\lambda I: \lambda \neq 0, \text{rational}\}$$

with

$$I = \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix},$$

and

$$F = SL(2, Z) / \{\pm I\}$$

act effectively on

$$H = \{z \in \mathbf{C}: \Im z > 0\}$$

by letting

$$g(z) = \frac{az + b}{cz + d}$$

for

$$g = \begin{pmatrix} a, & b \\ c, & d \end{pmatrix} \in GL^+(2, Q).$$

---

<sup>1</sup> Work supported by the National Research Council of Canada.