POTENTIAL THEORY FOR INFINITELY DIVISIBLE PROCESSES ON ABELIAN GROUPS[1]

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Let $\mathfrak{G}$ be a 2nd countable locally compact, noncompact Abelian group, and let X_t be an infinitely divisible process (henceforth called an i.d. process) with stationary increments taking values in $\mathfrak{G}$. The transition functions $P^t(x, dy)$ of such a process have the Feller property, so via a fundamental theorem in the theory of Markov processes (see [1, p. 44 ff.]) there is a realization of the process as a standard Markov process. Henceforth we shall assume that X_t is this version of the process. A point $x \in \mathfrak{G}$ is called *possible if* for each neighborhood N of 0 there is a $t>0$ such that $P^t(0, N+x)>0$. The set Σ of all possible points is a closed subsemigroup of $\mathfrak{G}$. Except when discussing the renewal theorem we will always assume that $\Sigma=\mathfrak{G}$. The process is called *recurrent if* $\int_0^\infty P^t(x, A)dt=\infty$ for all nonempty open sets A and all points x. Otherwise the process is called *transient*. In that case $\int_0^\infty P^t(x, A)dt<\infty$ for all compact sets A and all points x. A transient process is called *strongly transient* if $\int_0^\infty dt \int_t^\infty P^s(x, A)ds<\infty$ for all compact sets A and all points x. Otherwise the transient process is called *weakly transient*. In that case $\int_0^\infty dt \int_t^\infty P^s(x, A)ds=\infty$ for all nonempty open sets A and all points x. Let $f \geqq 0$ be a continuous function with compact support having integral 1, and set $r(t)=\int_t^\infty P^s f(0)ds$. Then a transient process is strongly transient if and only if $\int_0^\infty r(t)dt<\infty$. A process is called *nonsingular* if for some $t>0$ the distribution of X_t has a nonsingular component relative to Haar measure on $\mathfrak{G}$.

Very briefly, our purpose is to show that results in [3] and [4] for random walks on $\mathfrak{G}$ go over to i.d. processes on $\mathfrak{G}$, and this we do rather completely. These results are new even for i.d. processes on Euclidean spaces. Space does not permit a detailed description of all these results (much less their proofs) so we will only sketch what has been done. We point out here, however, that although most of the results on random walks have their counterpart for i.d. processes, the proofs of these results in many cases require significant new ideas. Some of these ideas involve indirect reductions to the discrete time case.

Let B be a relatively compact set, and let $T_B=\inf\{t \geqq 0: X_t \in B\}$ denote the first hitting time of B. We show that a theory of λ-capacities can be developed for *any* i.d. process. If the λ-capacity $C^\lambda(B)=0$ for one $\lambda>0$, then this is true for all $\lambda>0$, and then $P_x(T_B<\infty)=0$ a.e. On the other hand, if $C^\lambda(B)>0$ for some $\lambda>0$, then this is true

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