ON SINGULAR INTEGRALS

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The note is divided into three sections. The first section is devoted to singular kernels in R^n. Most of the results of the section remain valid after some modifications, if we replace R^n by a locally compact group and the Lebesgue measure by the Haar measure of the group; the second section deals with those extensions.

In the third section we apply the results of the first section to obtain L^p estimates for kernels whose homogeneity is given over a one parameter group. These kernels have been first considered by M. de Guzman [2]; particular cases of these kernels are those studied by A. P. Calderón and A. Zygmund in [1]; and by E. B. Fabes and N. Rivière in [3].

1. Singular kernels. Let $\{U_\alpha, \alpha>0\}$ be a family of open subsets of R^n, satisfying:

(a) $0\in U_\alpha$; for $\alpha<\beta$, $U_\alpha\subset U_\beta$; $\bigcap_\alpha U_\alpha=\{0\}$, the closure of U_α compact.

(b) There exists $\phi(\alpha)$ continuously mapping R_+ onto R_+ such that

$$U_\alpha - U_\alpha \subset U_{\phi(\alpha)} \quad\text{and}\quad m(U_{\phi(\alpha)}) \leqq Am(U_\alpha)$$
$$U_\alpha - U_\alpha = \{z; z = x - y, x \in U_\alpha, y \in U_\alpha\}.$$

(Clearly $\alpha<\phi(\alpha)$), $m(\cdot)$ denotes the Lebesgue measure.

(c) The function $f(\alpha)=m(U_\alpha)$ is left continuous and $f(\alpha)\to\infty$ as $\alpha\to\infty$.

We shall say that the operator T defined over a class of measurable functions is sublinear if

$$|T(f+g)| \leqq |T(f)| + |T(g)|,$$
$$L^p(R^n) = \left\{f; \|f\|_p = \left(\int_{R^n} |f(x)|^p dx\right)^{1/p} < \infty\right\}.$$

THEOREM 1 (WEAK TYPE). *Let $\{U_\alpha\}$ be a family as above, T a sublinear operator defined in $L^1(R^n)\oplus L^p(R^n)$ satisfying:*

(i) *For $f\in L^p(R^n)\cap L^\infty(R^n)$, $|Tf(x)|\leqq|T_1f(x)|+|T_2f(x)|$ where $m\{x; |T_1f(x)|>t\}\leqq(c/t^p)\int_{R^n}|f|^p dx$ and $\|T_2f\|_{L^\infty}\leqq\|f\|_{L^\infty}$.*

(ii) *If $f\in L^1(R^n)$ with support contained in $x+U_\alpha$, and if*

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