A NOTE ON SLIT MAPPINGS

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Communicated by Prof. Wolfgang H. Fuchs, February 24, 1969

1. **Introduction.** Recently the unitary properties of Grunsky's matrix have been studied by several authors. Milin [5] was apparently the first to observe these properties, and Pederson [6], unaware of Milin's work, rediscovered them independently later.

Let $f(z)=z+\sum_{k=2}^{\infty} a_k z^k$ be a regular univalent function in the unit circle. The function

$$\log \frac{f(z)-f(\zeta)}{z-\zeta} = \sum_{n,k=0}^{\infty} d_{nk} z^n \zeta^k$$

is then regular in $|z|<1$, $|\zeta|<1$.

Grunsky's matrix $B=(b_{nk})$, $b_{nk}=(nk)^{1/2} d_{nk}$, $n, k=1, 2, \cdots$ plays an important role in the theory of univalent functions; for example, simple proofs of the Bieberbach conjecture for $n=4$ were arrived at through its properties [2], [3].

If $1/f(z)=1/z+c_0+c_1z+\cdots$ maps $|z|<1$ onto a domain D such that the area (in the Lebesgue sense) of the complementary of D is zero—then Grunsky's matrix is unitary [5, Theorem 1], [6, Theorem 2.2]. As Milin pointed out, the area of the complementary of D is zero if and only if $\sum_{n=1}^{\infty} n|c_n|^2=1$. Following Pederson, these functions $f(z)$ will be referred as "slit mappings."

2. **Properties of slit mappings.** We now prove the following

THEOREM. *If* $f(z)=z+\sum_{n=2}^{\infty} a_n z^n$ *is a slit mapping then*

$$\frac{1}{f(z)} = \frac{1}{z} + c_0 + c_1 z + \cdots$$

either is of the form $1/z+c_0+c_1z$, $|c_1|=1$, *or there are infinitely many nonvanishing coefficients* c_k.

PROOF. The above theorem may also be formulated in the following way:

If $f(z)$ *is a slit mapping such that*

$$(1) \qquad \frac{1}{f(z)} = \frac{1}{z} + c_0 + c_1 z + \cdots + c_n z^n, \qquad c_n \neq 0,$$

then $n=1$ *and* $|c_1|=1$.