TRIANGULATING NONSIMPLY CONNECTED MANIFOLDS

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Lashof and Rothenberg have recently announced the following

THEOREM. *Let M^n be a compact topological manifold with boundary N^{n-1}, with fundamental group satisfying condition S.*

(a) *If $H^4(M; Z_2) = H^3(N; Z_2) = 0$, and $n \geqq 6$, M admits a PL manifold structure.*

(b) *If N already has a PL structure, $H^4(M; Z_2) = H^3(N; Z_2) = 0$ and $n \geqq 5$, then M admits a PL manifold structure agreeing with the given one on the boundary.*

The condition S is that $\pi_1(M \times T^k)$ and $\pi_1(\partial M \times T^k)$ satisfy the necessary conditions for the splitting theorems to hold, where T^k is the k-torus. If $\pi_1(M)$ and $\pi_1(\partial M)$ are free abelian, then condition S is satisfied. The purpose of this note is to relax the condition on the fundamental group.

THEOREM 1. *Let M^n be a closed, orientable topological manifold of dimension $n \geqq 7$ with $H^4(M; Z_2) = 0$. Then M has a PL structure.*

PROOF. By [1] and [2] or by [3], the stable homeomorphism conjecture is true in these dimensions, so M has a stable structure. By [4], $\pi_1(M)$ is generated by imbedded one spheres with product neighborhoods. Let f_i: $S_i^1 \times D^{n-1} \to M$ be such imbeddings, $i = 1, 2, \cdots, k$. We may assume that the $f_i(S_i^1 \times D^{n-1})$'s are disjoint. Let $0 < \alpha < 1$ and $D_\alpha^{n-1} = \{x \in R^{n-1} \mid \|x\| \leqq \alpha\}$. Henceforth we ignore the f_i and consider $S_i^1 \times D_\alpha^{n-1} \subset S_i^1 \times D^{n-1} \subset M$.

Let

$$\overline{M} = M - \bigcup_{i=1}^{k} (\operatorname{int}(S_i^1 \times D_\alpha^{n-1}))$$

and

$$M' = \overline{M} \cup \bigcup_{i=1}^{k} (D_i^2 \times S^{n-2}).$$

That is, perform surgery on M to kill $\pi_1(M)$. Then $\pi_1(M') = 0$ and $H^4(M'; Z_2) = 0$. By Lashof and Rothenberg, M' has a PL structure. Let $V = M - \bigcup_{i=1}^{k} (S_i^1 \times D_\alpha^{n-1}) = M' - \bigcup_{i=1}^{k} (D_i^2 \times S^{n-2})$.

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