A "FUNCTIONAL EQUATION" FOR MEASURES AND A GENERALIZATION OF GAUSSIAN MEASURES

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1. **Introduction.** Let G be a LCA group for which the map $x \mapsto 2x$ is an automorphism, and let $\xi: G\times G \to G\times G$ be defined by $\xi(x, y) = (x+y, x-y)$. We call a regular complex-valued measure μ on G Gaussian iff $\exists$ a second measure ν on G such that for all Borel sets $E\subseteq G\times G$,

$$(\mu \times \mu)(E) = (\nu \times \nu)(\xi(E)). \tag{1.1}$$

One rationale for this definition is that any finite probability measure on $\boldsymbol{R}$ which satisfies (1.1) is a Gaussian distribution with mean 0. (See [1, p. 77] for a proof.) Another reason is that the 2-adic theta functions defined by Mumford in [2] are related to 2-adic measures satisfying (1.1) much as ordinary theta functions are related to the Gaussian distribution $\exp(-ax^2)dx$.

Actually, we shall consider all set functions which are finite complex linear combinations of regular measures on G. These need not be σ-additive measures (since the regular measures need not be bounded), but we shall use the term measure for such functions as well.

The problem we consider is that of determining all Gaussian measures on G. In [2], Mumford did this in the case $G = (Q_2)^n$; in §§3 and 4 of this paper we state the results for $G = \boldsymbol{R}^n$ and for G a compact group. One reason for considering these cases is given by the following structure theorem.

THEOREM 1. *If G is a LCA group such that $x\mapsto 2x$ is an automorphism, then G can be written as $V\times W\times G_0$, where V is a real vector group, W is a 2-adic vector group, and G_0 contains a compact open subgroup for which $x\mapsto 2x$ is an automorphism.*

Another attack on the problem is considered in §2, where we consider Gaussian measures which are absolutely continuous (with respect to Haar measure). The rationale behind this approach is the following result.

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[1] The results announced here are contained in the author's Ph.D. thesis at Harvard University, written while he held an N.S.F. Graduate Fellowship.