ON EXISTENCE AND RIGIDITY OF ISOMETRIC IMMERSIONS

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Communicated by Edgar H. Brown, Jr., January 9, 1969

Introduction. In the study of the geometry of a hypersurface in euclidean space, the two fundamental results are the existence theorem and the rigidity theorem. The existence theorem states that a simply connected Riemannian manifold equipped with a second fundamental form for which the Gauss and Codazzi-Mainardi equations hold can be realized as a codimension one immersed submanifold of euclidean space. The rigidity theorem asserts (roughly) that any two such realizations differ by a rigid motion of the containing euclidean space. The effect of these two theorems is to reduce the study of immersed hypersurfaces in euclidean space to the study of Riemannian manifolds equipped with second fundamental forms satisfying the Gauss and Codazzi-Mainardi equations.

The purpose of this paper is to generalize these two results to isometric immersions of Riemannian manifolds in euclidean space with arbitrary codimension (always greater than zero). Our existence theorem is an analogue of the result of Hirsch [2] for smooth immersions. It states that a simply connected Riemannian manifold which has a k-plane bundle over it equipped with a bundle metric, a compatible connection, and a second fundamental form for which the Gauss and Codazzi-Mainardi equations hold can be isometrically immersed in euclidean space of codimension k. The rigidity theorem asserts that the normal bundle of an isometric immersion together with its induced bundle metric, connection, and second fundamental form essentially determine the immersion up to a rigid motion of the euclidean space.

It should be remarked here that the techniques used to prove the above results also apply to isometric immersions in spheres and hyperbolic spaces. Results of this type and detailed proofs of the results announced here will appear in [4].

Finally, I would like to express my gratitude to Rolph Schwarzenberger for helpful conversations during the early stages of this work.

[1] During the preparation of this paper, the author was partially supported by NSF grant GR 7993.