ON P^2-IRREDUCIBLE 3-MANIFOLDS[1]

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In [2] F. Waldhausen proved that two (compact) orientable 3-manifolds which are irreducible, boundary irreducible and sufficiently large, are homeomorphic iff there exists an isomorphism between the fundamental groups which respects the peripheral structure. This is Corollary (6.5) to his Theorem (6.1) in [2]. We extend this theorem to apply to nonorientable 3-manifolds.

Throughout this paper a "manifold" M is a compact 3-manifold. A "surface F in M" is a properly embedded connected 2-manifold in M (i.e. $F \cap \delta M = \delta F$).

1. The main theorem.

DEFINITION. (1) A manifold M is P^2=irreducible iff M is irreducible and does not contain 2-sided projective planes (properly embedded).

(2) M is boundary irreducible iff δM is incompressible (see definition in [2]).

THEOREM A. *Let M, N be P^2-irreducible and boundary irreducible. If M is orientable and closed let M be sufficiently large. Let $\pi_1(N) \neq 1$ and suppose f: $(N, \delta N) \to (M, \delta M)$ is a map such that*

$$\ker(f_*: \pi_1(N) \to \pi_1(M)) = 0.$$

Then there exists a homotopy f_γ: $(N, \delta N) \to (M, \delta M)$, $\gamma \in I$, of $f = f_0$ such that either (a) *or* (b) *holds.*

(a) *f_1: $N \to M$ is a covering map.*

(b) *N is a line bundle over a closed surface and $f_1(N) \subset \delta M$.*

If $f/\delta N$ is locally homeomorphic, then f_γ may be chosen constant on δN.

COROLLARY. *Let M, N be P^2-irreducible and boundary irreducible. If M is orientable and closed let M be sufficiently large. Suppose σ: $\pi_1(N) \to \pi_1(M)$ is an isomorphism which respects the peripheral structure. Then there exists a map f: $N \to M$ which induces σ and such that either* (a) *or* (b) *holds.*

(a) *f: $N \to M$ is a homeomorphism.*

(b) *M is the product line bundle over a closed surface F and N is a twisted line bundle over F and f: $(N, \delta N) \to (M, \delta M)$ contracts into δM.*

[1] These results were obtained while the author was a DAAD-fellow at the University of Illinois in 1967/68. Full details will appear in the author's thesis at Rice University.