INFINITE-DIMENSIONAL MANIFOLDS ARE OPEN SUBSETS OF HILBERT SPACE

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In this paper we prove, using Hilbert space microbundles, the

THEOREM. *If M is a separable metric manifold modeled on the separable infinite-dimensional Hilbert space, H, then M can be embedded as an open subset of H.*

Each infinite-dimensional separable Fréchet space (and therefore each infinite-dimensional separable Banach space) is homeomorphic to H. (See [1].) We shall use "F-manifold" to denote "metric manifold modeled on a separable infinite-dimensional Fréchet space." Thus we have

COROLLARY 1. *Each separable F-manifold can be embedded as an open subset of H.*

Recent results of Eells and Elworthy [6] and Kuiper and Burghelea [9] and Moulis combine to show (see [6]) that two separable C^∞ Hilbert manifolds are C^∞-diffeomorphic if and only if they have the same homotopy type. Since open subsets of H have an induced C^∞ structure, we have

COROLLARY 2. *Each F-manifold has a unique C^∞-Hilbert manifold structure.*

COROLLARY 3. *Two F-manifolds are homeomorphic if and only if they have the same homotopy type.*

Results about open subsets of H in [7] apply to give us

COROLLARY 4. *For each F-manifold M there is a countable locally-finite simplicial complex K, such that M is homeomorphic to $|K|\times H$.*

COROLLARY 5. *Each F-manifold is homeomorphic to an open set $U\subset H$, such that $H-U$ is homeomorphic to H and* $\mathrm{bd}(U)$ *is homeomorphic to U and to* $\mathrm{cl}(U)$.

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1. Hilbert microbundles. Milnor, in [11], defined the concept of (Euclidean) microbundles. We shall use strongly the ideas and defini-

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