SOME INEQUALITIES FROM SWITCHING THEORY[1]

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The following inequality was discovered by Turner and Conway (SIAM Rev. **10** (1968) 107).

Let $0<p<1$, $0<q<1$, $p+q=1$, $m>1$, $n>1$. Define $F(p, q) = (1-p^m)^n+(1-q^n)^m-1$. Then $F(p, q)>0$. Their derivation is based on reliability theory. The following variants and generalizations are given. First, the inequality is reversed if $0<m$, $n<1$. Second, the two finer estimations

$$F(p, q) \geqq \sum_r \binom{n}{r} p^{mn-n-mr+2r}q^{n+mr-2r},$$

$$\left[1 - p^m - \binom{m}{1} qp^{m-1} - \cdots - \binom{m}{r} q^r p^{m-r}\right]^n + [1 - q^n]^m$$
$$+ \binom{m}{1} q^n(1 - q^n)^{m-1} + \cdots + \binom{m}{r} q^{nr}(1 - q^n)^{m-r} > 1$$

hold $(0<r<n)$. Next, let p_i, m_i be a set of k positive numbers; let r be nonnegative and suppose $n\geqq 1$. Suppose further $\forall_i\{m_i>1\}$, $n\cdot\prod m_i=S$, $r+\sum p_i=1$. Then if $r=0$, $n=1$, we have

$$\sum (1 - p_i^{S/m_i})^{m_i} > k - 1.$$

If $r>0$, $n>1$ (and in certain other cases) we have

$$\sum \{1 - (1 - p_i)^{S/m_i}\}^{m_i} > (1 - r^n)^{S/n}.$$

Contrary to the proof given by reliability theory, the combinatorial proofs of this paper are symmetric in the parameters. Independent *analytic* proofs involve induction, together with estimation of the minimum of a nonconvex function. This article will appear in Journal of Combinatorial Theory.

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