## RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable.

# ON PROJECTIONS OF SELFADJOINT OPERATORS AND OPERATOR PRODUCT ADJOINTS

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**Introduction.** The proofs given in §§1 and 2 of this note were contained in a letter from the present writer to G. Lumer, sent shortly after the appearance of [**1**]. The subsequent appearance of [**2**] makes it desirable to publish these demonstrations. In [**3**] we mention an observation relevant to the topic of [**2**].

1. On [**1**]. [**1**] consists primarily of the proof of the following lemma and an addendum. Let $T$ be any selfadjoint operator in $\mathcal{H}$, a Hilbert space, $\mathcal{Y}$ a closed subspace in $\mathcal{H}$, $P$ the orthogonal projection of $\mathcal{H}$ onto $\mathcal{Y}$, $\chi=\mathcal{Y}^{\perp}$, the orthogonal complement of $\mathcal{Y}$ in $\mathcal{H}$. Let $T_0=PTP$.

LEMMA 1. *If $\chi$ is finite dimensional, then $T_0$ is selfadjoint.*

OUR PROOF. $\mathfrak{D}(T_0)=\mathfrak{D}(TP)$ is dense by [**3**, Theorem IV 2.7(iv), p. 103; note $\mathfrak{D}(T)$ dense is the only property of $T$ used in (iv)]. Any time $\mathfrak{D}(TP)$ is dense, $(T_0)^*\equiv[P(TP)]^*=(TP)^*P\supseteq PTP=T_0$, and $T_0$ is selfadjoint iff $(TP)^*=PT$ iff $PT$ is closed, the latter implication seen from $(TP)^*\equiv(T^*P^*)^*=(PT)^{**}\supseteq PT$, equality holding iff $PT$ is closed. But $PT$ is closed by [**3**, Theorem IV 2.7(i)].

The addendum of [**1**] asserts that a remark credited to G. Lumer in the acknowledgment of [**4**] is incorrect. However, the remark is correct in the context of *bounded* operators, which clearly was the context intended by Williams [**4**]. In that context, $T_0=ATA$ is obviously selfadjoint whenever $A$ and $T$ are.

2. **On** [**2**]. Let $S$ and $T$ be densely defined closed linear operators in $\mathcal{H}$; [**2**] deals with the interesting question of when $(TS)^*=S^*T^*$

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