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ON SPHERE-BUNDLES. I

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Let E be an $(n-1)$-sphere bundle over a base space B, with the orthogonal group as structural group. By an *almost-complex structure* on E we mean a reduction of the structural group to the unitary group. By an *A-structure* on E I mean a fibre-preserving map $f: E\to E$ such that fx is orthogonal to x for all $x\in E$. For example, an almost-complex structure determines such a map through the action[2] of the scalar J such that $J^2=-1$. Note that n must be even if an A-structure exists. When E is trivial this necessary condition is also sufficient.

I describe E as *homotopy-symmetric* if $1\simeq u: E\to E$, by a fibre-preserving homotopy, where u denotes the antipodal map given by $ux=-x$. This condition also implies that n is even. An A-structure f on E determines a fibre-preserving homotopy f_t $(t\in I=[0, 1])$, where $f_tx=x\cos \pi t+f(x)\sin \pi t$, and so E is homotopy-symmetric. I assert that the converse holds in the stable range,[3] so that we have

THEOREM 1. *Let B be a finite complex such that* $\dim B\leqq n-4$. *Then E admits an A-structure if and only if E is homotopy-symmetric.*

A proof can be given as follows. Let $p: E\to B$ denote the fibration. Let E' denote the space of pairs (x, y), where $x, y\in E$, such that $px=py$ and such that x is orthogonal to y. We fibre E' over E with projection p' given by $p'(x, y)=x$. An A-structure f on E determines a cross-section $f': E\to E'$, where $f'x=(x, fx)$, and conversely a cross-section determines an A-structure. Let E'' denote the space of paths λ in E such that $p\lambda$ is stationary in B and such that $\lambda(0)=\lambda(1)$. We

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[2] We recall that the centre of the structural group acts on the bundle.

[3] The stable range, in relation to this problem, is not quite as extensive as the stable range of ordinary theory.