A NEW CHARACTERIZATION OF $AR(\mathfrak{M})$ AND $ANR(\mathfrak{M})$

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The $AR(\mathfrak{M})$ spaces have been characterized as the r-images of convex sets in normed linear spaces, and the $ANR(\mathfrak{M})$ spaces as the r-images of open subsets of such sets. (The r-images of a space correspond to the retracts of the space. See Borsuk [1] for proofs of most of the results referred to here.) In this paper we present new characterizations of these spaces. The characterizations yield some interesting new results, as well as clarifications and new proofs of older results.

1. **The characterizations.** The fundamental notion is that of a guiding function, first defined (in the case of separable spaces) by Wojdyslawski [3].

DEFINITIONS. A *guiding function* for a metrizable space X is a continuous mapping g from a CW-polytope P into X such that

(i) every finite set S of vertices of P determines a simplex (denoted by conv$S$) in P;

(ii) g maps the vertices of P onto a dense subset of X;

(iii) if each S_n, $n=1, 2, \cdots$ is a finite set of vertices of P and $\lim_{n\to\infty} g(S_n)=x$ then $\lim_{n\to\infty} g(\text{conv}S_n)=x$.

A *locally guiding function* for a metrizable space is a mapping g which satisfies every condition for a guiding function except (i), which is replaced by

(i)′ Every $x\in X$ has a neighborhood W_x with the following property: every finite set S of vertices of P such that $g(S)\subset W_x$ determines a simplex conv$S$ in P.

THEOREM 1. *A metrizable space is an $AR(\mathfrak{M})$ if and only if it has a guiding function.*

THEOREM 2. *A metrizable space is an $ANR(\mathfrak{M})$ if and only if it has a locally guiding function.*

For a proof of Theorem 1 in the case of separable metric spaces see [3] and [2]. In the more general case a guiding function for an $AR(\mathfrak{M})$ space X is constructed as follows. We may assume X is a closed subset of a convex set K in a normed linear space and $r: K\to X$ is a retraction. If Z is any dense subset of X then in the real linear space R^Z we pick out the linearly independent set $V=\{v_z\in R^Z \mid z\in Z\}$, where $v_z(z')$ is 1 if $z=z'$ and 0 otherwise. Let $P=\text{conv}\,V$ (convex hull