9. S. Takahashi, *A duality theorem for representable locally compact groups with compact commutator subgroup*, Tôhoku Math. J. 2 (1952), 115–121.

10. E. Thoma, *Über unitäre Darstellungen abzählbar, diskreter Gruppen*, Math. Ann. 153 (1964), 111–138.

UNIVERSITY OF WASHINGTON, SEATTLE, WASHINGTON 98105

TWO SIDED IDEALS OF OPERATORS

BY HORACIO PORTA

Communicated by Paul Halmos, September 30, 1968

1. Let X be a Banach space, and $B(X)$ the Banach algebra of all bounded linear operators in X. The closed two sided ideals of $B(X)$ (actually, of *any* Banach algebra) form a complete lattice $L(X)$. Aside from very concrete cases, $L(X)$ has not yet been determined; for instance, when $X=l^p$, $1\leqq p<\infty$, $L(X)$ is a chain (i.e., totally ordered) with three elements: $\{0\}$, $B(X)$ and the ideal $C(X)$ of compact operators (see [3]). On the other hand, it is known [2, 5.23] that for $X=L^p$, $1<p<\infty$, the lattice $L(X)$ is *not* a chain. A treatment for X a Hilbert space of arbitrary dimension can be found in [4]. We aim to exhibit here a Banach space X such that $L(X)$ is both "long" and "wide." Precisely, we have

PROPOSITION. *There exists a real Banach space X with the properties:*

(i) *X is separable, isometric to its dual X^*, and reflexive;*

(ii) *it is possible to assign a closed two sided ideal $\mathfrak{a}(\mathfrak{F})\subset B(X)$ to each finite set of positive integers $\mathfrak{F}$, in such a way that the mapping $\mathfrak{F}\to\mathfrak{a}(\mathfrak{F})$ is injective and inclusion preserving in both directions: $\mathfrak{F}\subseteq\mathfrak{G}$ if and only if $\mathfrak{a}(\mathfrak{F})\subseteq\mathfrak{a}(\mathfrak{G})$.*

The example is described below, in §3.

2. In the sequel, all Banach spaces are *real* (the complex case can be dealt with similarly). If X, Y are Banach spaces, $\mathfrak{m}(Y, X)$ denotes the set of operators $T\in B(X)$ that can be factorized through Y, i.e., such that $T=SQ$ for suitable bounded linear operators $Q: X\to Y$, $S: Y\to X$. If Y is isomorphic (as a Banach space) to its square $Y\times Y$ ($\times$ means cartesian product), then (see [6, Proposition 1.2] or [2, Theorem 5.13]) $\mathfrak{m}(Y, X)$ is a two sided ideal of $B(X)$. $\mathfrak{a}(Y, X)$ will denote the (uniform) closure of $\mathfrak{m}(Y, X)$; thus, if Y is isomorphic to $Y\times Y$, $\mathfrak{a}(Y, X)$ is a *closed two sided ideal* of $B(X)$.

In all that follows, *subspace* means closed lineal subspace; a sub-