A NOTE ON THE STRUCTURE OF MOORE GROUPS

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1. **Introduction.** A locally compact group G will be called a Moore group if every continuous irreducible unitary representation of G is finite dimensional. Let [Moore] denote the class of all Moore groups, and let [Z] denote the class of all locally compact groups such that $G/Z(G)$ is a compact group, where $Z(G)$ denotes the center of G. S. Grosser and M. Moskowitz introduced the classes [Moore] and [Z], and made considerable progress on unifying and organizing the study of various "compactness conditions" in locally compact groups. (See [2], [3], and [4].) Grosser and Moskowitz have shown that $[Z]\subset[\text{Moore}]$, [3, Theorem 2.1, p. 369], and C. C. Moore has recently shown that $G\in[\text{Moore}]$ implies that G is an inverse limit of finite extensions of groups $H_\alpha\in[Z]$ (see Theorem 3A below). Other results on Moore groups are obtained below by introducing the notion of Takahashi groups. Let [Tak] denote the class of all locally compact groups G such that the derived group G' has compact closure, and G is maximally almost periodic, i.e., there exists a monomorphism from G into a compact group. The main results can be stated as follows:

THEOREM 1. *A group G satisfies $G\in[\text{Moore}]$ if and only if G contains a characteristic subgroup H such that H has finite index in G and $H\in[\text{Tak}]$.*

THEOREM 2. *A group G satisfies $G\in[\text{Moore}]$ if and only if G is a semidirect product $G=R^n\times_\phi B$, where $B\in[\text{Moore}]$ has a compact identity component B_e, and B contains a normal subgroup H with finite index such that $R^n\times_\phi H$ is a direct product $R^n\times H$.*

Theorem 2 may be interpreted as a type of generalized Freudenthal-Weil theorem (see Theorem 3C below). Consequences of Theorem 1 are that quotient groups of Takahashi groups are Takahashi groups, and (closed) subgroups of Moore groups are Moore groups. (This behavior is a pleasant contrast to results such as the following:

(1) Closed subgroups of [Z]-groups need not be [Z]-groups.

(2) G/H need not be in [MAP] even when $G\in[\text{MAP}]$ and H is a closed characteristic subgroup of G.)

It follows that the class [Moore] is stable under subgroups, quotient groups, inverse limits, and finite extensions; hence the class