MAXIMAL ABELIAN SUBALGEBRAS IN HYPERFINITE FACTORS

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1. **Introduction.** In this note we outline a construction which, together with a new invariant, gives still more maximal abelian subalgebras of the hyperfinite factor of type II_1. We also state some results concerning maximal abelian subalgebras of hyperfinite factors of type III. Complete proofs will appear elsewhere.

First we will establish some notation and terminology. Let $\mathfrak{M}$ and $\mathfrak{N}$ be maximal abelian subalgebras of a factor $\mathfrak{A}$. We call $\mathfrak{M}$ and $\mathfrak{N}$ equivalent (in $\mathfrak{A}$) if there is an automorphism of $\mathfrak{A}$ carrying $\mathfrak{M}$ onto $\mathfrak{N}$. Let $N(\mathfrak{M})=N^1(\mathfrak{M})$ be the subalgebra of $\mathfrak{A}$ generated by all those unitary operators U in $\mathfrak{A}$ with $U\mathfrak{M}U^*=\mathfrak{M}$, and let $N^j(\mathfrak{M})=N(N^{j-1}(\mathfrak{M}))$ for $j>1$. Following Dixmier [2] and Anastasio [1], we call $\mathfrak{M}$ regular if $N(\mathfrak{M})=\mathfrak{A}$, semiregular if $N(\mathfrak{M})$ is a factor distinct from $\mathfrak{A}$, and n-semiregular ($n\geqq 2$) if no $N^j(\mathfrak{M})$, $1\leqq j<n$ is a factor but $N^n(\mathfrak{M})$ is. We say that $\mathfrak{M}$ has proper [improper] length n if n is the smallest positive integer such that $N^n(\mathfrak{M})=N^{n+1}(\mathfrak{M})$ and if $N^n(\mathfrak{M})=\mathfrak{A}$ [$N^n(\mathfrak{M})\neq\mathfrak{A}$]. Notice that the length of a maximal abelian subalgebra is invariant under equivalence. We are now able to state our main results.

THEOREM 1. *For each choice of* $n=2, 3$ *and* $k=0, 1, 2, \cdots$, *the hyperfinite* II_1 *factor contains an n-semiregular maximal abelian subalgebra of improper length* $n+k$.

THEOREM 2. *Let* $\mathfrak{A}$ *be one of the hyperfinite type* III *factors of Powers (cf.* [5]*). Then* $\mathfrak{A}$ *contains a regular and two inequivalent semiregular maximal abelian subalgebras. Also, for each choice of* $n=2, 3$ *and* $k=0, 1, 2, \cdots$, $\mathfrak{A}$ *contains two n-semiregular maximal abelian subalgebras, one of proper length* $n+k$ *and one of improper length* $n+k$.

REMARKS. (1) These results are a summary of the author's doctoral thesis written at the University of British Columbia under the supervision of Dr. D. Bures.

(2) Anastasio has shown that Theorem 1 holds with "improper" replaced by "proper" [1]. Because of a previous remark, our subalgebras are mutually inequivalent as well as inequivalent to those of Anastasio.

(3) $\mathfrak{M}$ has proper length n if and only if $\mathfrak{M}$ has length $n-1$ in the sense of Tauer [7].