A CONVOLUTION EQUATION AND HITTING PROBABILITIES OF SINGLE POINTS FOR PROCESSES WITH STATIONARY INDEPENDENT INCREMENTS

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Communicated by Henry P. McKean, Jr., November 19, 1968

In this note we study the hitting probability of a point r,

$$h(r) = P\{X_t = r \text{ or } X_{t-} = r \text{ for some } 0 < t < \infty\},$$

for a d-dimensional right continuous process $\{X_t\}_{t\geq 0}$ with stationary independent increments and $X_0 = 0$ with probability 1. It is well known (see [7, §62], [1, §3.4], or [5, §4]) that the characteristic function for any such process is of the form

$$(1)\quad Ee^{i\lambda\cdot X_t} = \exp\left\{i\lambda\cdot at - \tfrac{1}{2}Q(\lambda)t + t\int\left[e^{i\lambda\cdot y} - 1 - \frac{i\lambda\cdot y}{1+|y|^2}\right]\nu(dy)\right\},$$

$\lambda = (\lambda_1, \cdots, \lambda_d) \in R^d$, $a \in R^d$, Q a nonnegative definite quadratic form and ν a Borel measure on $R^d - \{0\}$ for which

$$\int \min(1, |y|^2)\nu(dy) < \infty.$$

Our purpose is to determine when $h(r)$ is strictly positive, respectively zero. An old and obvious result is that $h(r) > 0$ for all r if X_t is Brownian motion. Somewhat more difficult is the behavior of $h(r)$ for symmetric stable processes of index α in dimension one. For such processes it was shown by Lévy, Erdös and Kac (see [6]; also [12] for simplified proofs) that $h(r) > 0$ for all $r \in R$ if $1 < \alpha \leqq 2$ and $h(r) = 0$ for all $r \in R$ if $\alpha \leqq 1$. It will be seen that this result is typical for the general situation as well. The motivation for our study was a problem of Chung's [2], [3], which asked to solve the one-dimensional convolution equation

$$(2)\qquad \int_{0-}^{r+} \sigma(r-s)W(ds) = 1, \qquad r > 0.$$

Here σ is a given decreasing right continuous function on $[0, \infty)$ such

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[1] Research supported by the NSF under grant GP 7128.