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FUNCTIONS OF BOUNDED CONVEXITY

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1. **Introduction.** Functions of bounded variation on $[a, b]$ are those functions for which

$$(1) \qquad V_a^b(f) = \operatorname*{Sup}_P V(f, P) = \operatorname*{Sup}_P \sum_{j=1}^{n} |\Delta f_j|$$

is finite. An important theorem about the set $BV[a, b]$ of all such functions says that this set may be characterized as the set of all functions representable as the difference of two nondecreasing functions. Stated with less precision but more suggestion for our purposes, $BV[a, b]$ is the set of all functions representable as the difference of two functions with nonnegative first derivatives. It is then natural to consider the set of all functions representable as the difference of two functions with nonnegative second derivatives (convex functions, roughly speaking).

We begin our study with an expression that plays the role of (1). For a partition $P=\{a=x_1<x_2<\cdots<x_n=b\}$, let $\Box f_j = [f(x_j)-f(x_{j-1})/(x_j-x_{j-1})]$.

DEFINITION 1. For f: $[a, b]\to R$, let

$$(2) \qquad K_a^b(f) = \operatorname*{Sup}_P K(f, P) = \operatorname*{Sup}_P \sum_{j=1}^{n-1} |\Box f_{j+1} - \Box f_j|.$$

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