

THE PSEUDO-CIRCLE IS NOT HOMOGENEOUS¹

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1. Introduction. Two well-known and hitherto unsolved problems concerning limit spaces are those raised by R. H. Bing in 1951 in [2, p. 49]. These problems, which have also been discussed at the Summer Institute on Set Theoretic Topology of the American Mathematical Society, Madison, Wisconsin, 1955 by F. B. Jones [9], are the questions of whether or not all pseudo-circles are topologically equivalent and whether or not there exists a homogeneous pseudo-circle. In a recently completed paper [8], a research announcement of which was presented in [7], this author has given an affirmative solution to the first of these two problems. The purpose of this present paper is to give a complete solution to the second and more widely discussed problem.

2. Preliminaries. Throughout this paper we shall use terms and notations which are either the same as, or natural extensions of those developed by this author in [4], [5], [6] and [8]. In general these concepts were originally suggested by those used by Bing in [1], [2] and [3]. For convenience of reference we give statements of the definitions of the less standardized of these special terms that are needed in this paper. In the presentation of these definitions and in the development of the results of this paper it will be assumed that all circular chains have at least six links and the domains and ranges of all cyclic r -patterns have at least six elements. This will allow us to avoid the formal awkwardness of treating cases where the circular configurations and winding or crookedness characteristics are trivial, and will not involve any loss in generality in establishing the principal results of this paper.

The terms *r-pattern*, *cyclic r-pattern*, *winding number* and *linear representation* are defined in [5]. In this paper we make a minor extension of the definition of the first of these terms by allowing the domain of an r -pattern to be any set of consecutive integers, not necessarily a finite set. The r -pattern associated with a normal refine-

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