SOME NONZERO HOMOTOPY GROUPS OF SPHERES

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1. The purpose of this note is to establish some nonzero elements in the homotopy groups of spheres. This results from unstabilizing a method of Adams. Namely, an Adams spectral sequence is used to detect elements in $\pi_{n+i}(S^n)$ for various n and i; in addition to the d and e invariants of Adams, the Hopf invariants are used to show that certain of these elements are nonzero. One consequence will be the following.

Consequence. The groups $\pi_{4+i}(S^4)$ are nonzero for all $i \geqq 0$.

2. Recall the mod-p-restricted lower central series spectral sequence (abbr: mod-p-RLCSSS), constructed as in [4], [5] and [10]. For each simplicial set X, form GX as in [6], filter GX by its mod-p-RLCS, and pass to the homotopy exact couple. The resulting spectral sequence we will label $E^r_{s,d}(X)$, where s = filtration and d = dimension. The results of [4, §(2.4)] show that for the sphere spectrum S, the term $E^1(S)$ of the mod-2-RLCSS is a ring Λ, with multiplicative generators λ_i for each $i \geqq 0$. An additive basis for $E^1(S)$ consists of all monomials $\lambda_I = \lambda_{i_1} \cdots \lambda_{i_k}$, where $I = (i_1, \cdots, i_k)$ is a sequence of nonnegative integers with $2i_j \geqq i_{j+1}$ for $j = 1, 2, \cdots, k-1$. Call such monomials allowable. In the unstable case, the results of [4, §(5.4)] show that for the n-sphere S^n, $E^1(S^n)$ is the subvector space of Λ with basis all λ_I which are allowable and for which $i_1 < n$. Such a monomial $\lambda_I \in E^1(S^n)$, where $I = (i_1, \cdots, i_k)$, has filtration k, and dimension $n + \sum i_j$.

3. There is a short exact sequence of differential vector spaces:

$$0 \to E^1_{s,n+i}(S^n) \xrightarrow{i} E^1_{s,n+i+1}(S^{n+1}) \xrightarrow{h} E^1_{s-1,n+i+1}S^{2n+1}) \to 0$$

where i is the inclusion and h is defined on the allowable basis by

$$\begin{aligned} h(\lambda_j \lambda_I) &= \lambda_I \quad \text{for } j = n, \\ &= 0 \quad \text{for } j < n. \end{aligned}$$

From this, there derives a long exact sequence

$$(3.1) \qquad \cdots \to E^2(S^n) \xrightarrow{i_*} E^2(S^{n+1}) \xrightarrow{h_*} E^2(S^{2n+1}) \xrightarrow{\partial} \cdots.$$

It can be shown that h_* commutes with all differentials, and is induced