IMMERSIONS AND SURGERIES OF TOPOLOGICAL MANIFOLDS

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In this announcement, we outline a version of Haefliger and Poenaru's Immersion Theorem [2] for topological manifolds. We then use our theorem to do surgery on topological manifolds, and obtain results such as the following: Let M^n be a closed, almost parallelizable topological manifold (that is, the tangent bundle of $M-p$ is trivial, $p \in M$) which has the homotopy type of a finite complex. Then by a sequence of surgeries, M can be reduced to an $[n/2-1]$ connected almost parallelizable manifold.

In order to state the Immersion Theorem we give the following definitions: Let M, M' and Q be topological manifolds, M a compact locally flat submanifold of the open manifold M', with $\dim M' = \dim Q$.

Write $\mathrm{Im}_{M'}(M, Q)$ for the semisimplicial complex of M' immersions of M in Q; a simplex of $\mathrm{Im}_{M'}(M, Q)$ is an immersion $f: \Delta \times U \to \Delta \times Q$ commuting with the projections on the standard simplex Δ, where U is a neighborhood of M in M'. Two such are identified if they agree on $\Delta \times$ (a neighborhood of M in M').

Write $R(TM' | M, TQ)$ for the semisimplicial complex of representation germs of the tangent bundle of M' restricted to M in the tangent bundle of Q; a simplex of $R(TM' | M, TQ)$ is a microbundle map Φ of $\Delta \times TU$ in $\Delta \times TQ$ which commutes with projections on Δ, U a neighborhood of M in M', such that the map of $\Delta \times TU$ in $\Delta \times U \times Q$ given by $(t, u, u') \to (t, u, \pi\Phi(t, u, u'))$ is an immersion on a neighborhood of $\Delta \times$ (the diagonal of M). Two such representations define the same representation germ if they agree on a neighborhood of $\Delta \times$ (the diagonal of M.

Observe that if f is a simplex of $\mathrm{Im}_{M'}(M, Q)$, the map df defined as follows, is a simplex of $R(TM' | M, TQ)$: $df(t, u, u') = (t, f_t u, f_t u')$ where $u, u' \in U$, $f(t, u) = (t, f_t u)$. We now state the Immersion Theorem. Suppose M has a handlebody decomposition with all handles of index $<\dim Q$. Then the map $d: \mathrm{Im}_{M'}(M, Q) \to R(TM' | M, TQ)$ is a homotopy equivalence. R. Lashof has shown [6] that the hypothesis

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