# SMOOTH HOMOTOPY PROJECTIVE SPACES

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**Introduction.** In [5] we considered certain fixed point free involutions on Brieskorn manifolds as weakly complex bordism elements. In [4] we considered associated examples of smooth normal invariants for real projective spaces, settling the realizability question for dimensions $\not\equiv 1 \bmod 4$ and the desuspendability question for dimensions $4k+1$. The object of this study is the classification of these smooth normal invariants given by the Brieskorn examples. Our results overlap somewhat with Atiyah and Bott [2] as well as Browder [3], but our methods are entirely different and our results rather more refined. Full details of these and related results will appear elsewhere.

1. **Smooth normal invariants.** Following Sullivan [6], we regard a smooth normal invariant of a space $X$ as an element of $[X, G/O]$. Of course, we have $G/O \cong SG/SO$. We need the fibers $SG/\mathrm{Spin}$ of $B\mathrm{Spin} \to BSG$ and $SO/\mathrm{Spin} \simeq P^\infty$ of $B\mathrm{Spin} \to BSO$. The spaces $SG/SO$, $SG/\mathrm{Spin}$, $SO/\mathrm{Spin}$ have their Whitney $H$-space structures under which the sequence

$$SO/\mathrm{Spin} \to SG/\mathrm{Spin} \to SG/SO$$

is a multiplicative fibration.

A map $\mu$: $SG/\mathrm{Spin} \to BO$ is constructed as follows. Let $\gamma_n$ denote the universal fiber space over $BSG_n$ with fiber $S^{n-1}$, $\beta_n$ the pullback to $B\mathrm{Spin}_n$, and $\alpha_n$ the pullback to $SG_n/\mathrm{Spin}_n$; also, let $\epsilon_n$ denote the $S^{n-1}$ fibration over a point. Corresponding to the commutative diagram

$$\begin{array}{ccccc} & \nearrow & B\mathrm{Spin}_n & \searrow & \\ SG_n/\mathrm{Spin}_n & & & & BSG_n \\ & \searrow & \{pt\} & \nearrow & \end{array}$$

of spaces, there is the commutative diagram

$$\begin{array}{ccccc} & \nearrow & \beta_n & \searrow & \\ \alpha_n & & & & \gamma_n \\ & \searrow & \epsilon_n & \nearrow & \end{array}$$

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