ON THE AUTOMORPHISM GROUP OF A SEMISIMPLE JORDAN ALGEBRA OF CHARACTERISTIC ZERO

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Introduction. Let $\mathfrak{J}$ be a semisimple Jordan algebra over an algebraically closed field Φ of characteristic zero, and let G be the automorphism group of $\mathfrak{J}$. The purpose of this note is to present general results on G, the proofs of which do not involve the use of the classification theory of simple Jordan algebras over Φ. Specifically, we wish to determine the algebraic components $G_0, G_1, G_2, \cdots$ of the linear algebraic group G. To this end, we will give a formula for the number of components of G in terms of certain root-spaces associated with $\mathfrak{J}$ (see the Corollary to Theorem 3 and Theorem 6 below). For each component G_i of G, the index of G_i is defined to be the minimum dimension of the 1-eigenspaces of the automorphisms belonging to G_i. We will give a formula for the index of each component G_i of $\mathfrak{J}$ (see Theorem 8). Finally, we will give a table which applies these theorems to each of the simple Jordan algebras over Φ.

These results are analogous to those on Lie algebras given in [4, Chapter 9] and [5].

1. Notation and terminology. Let $\mathfrak{J}$, G, and Φ be as above. Following [3], we write $x.y$ for the product of elements x, y of $\mathfrak{J}$ and let $R_y: x \rightarrow x.y$. We let $\mathfrak{D}$ be the derivation algebra of $\mathfrak{J}$. We denote by $\mathfrak{L}$ the structure Lie algebra $R_{\mathfrak{J}} \oplus \mathfrak{D}$ of $\mathfrak{J}$, and by $\mathfrak{K}$ the Koecher-Tits algebra $\mathfrak{J} \oplus \bar{\mathfrak{J}} \oplus \mathfrak{L}$ of $\mathfrak{J}$ [3, Chapter 8]. $\mathfrak{D}$ and $\mathfrak{L}$ are completely reducible. Thus if $\mathfrak{C}$ is the center of $\mathfrak{L}$ and $\mathfrak{E}$ the center of $\mathfrak{D}$, then $\mathfrak{L} = \mathfrak{C} \oplus \mathfrak{L}'$ and $\mathfrak{D} = \mathfrak{E} \oplus \mathfrak{D}'$, where $\mathfrak{L}'$ and $\mathfrak{D}'$ are semisimple. $\mathfrak{K}$ is semisimple and is simple if and only if $\mathfrak{J}$ is simple. Let Γ be the structure group of $\mathfrak{J}$ [3, Chapter 2]. G and Γ are linear algebraic groups; we let G_0 and Γ_0 be respectively the algebraic components of the identity of these groups. If $\eta \in \Gamma$, then $\tilde{\eta}: a + \bar{b} + L \rightarrow a\eta + (b\eta^{\#-1})^- + \eta^{-1}L\eta$ is an automorphism of $\mathfrak{K}$; here $\eta^{\#} = U_{1\eta}\eta^{-1}$, where in general $U_x = 2R_x^2 - R_{x^2}$ (see [6]). The mapping $\eta \mapsto \tilde{\eta}$ is a birational isomorphism from Γ onto a subgroup $\tilde{\Gamma}$ of Aut $\mathfrak{K}$. $\tilde{\Gamma}$ is the subgroup of Aut $\mathfrak{K}$ of elements fixing R_1 (where 1 is the identity of $\mathfrak{J}$).

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