ON THE NONLINEAR EQUATIONS $\Delta u+e^u=0$ AND $\partial v/\partial t=\Delta v+e^v$

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1. **Introduction.** This note is concerned with the boundary value problem BVP for the equation

$$\Delta u + e^u = 0 \qquad (x \in \Omega) \tag{1}$$

under the boundary condition $u|_{\partial\Omega}=0$, and also with the initial value problem IVP for the equation

$$\partial v/\partial t = \Delta v + e^v \qquad (t \geqq 0,\ x \in \Omega) \tag{2}$$

under the initial condition $v|_{t=0}=a(x)$ and the boundary condition $v|_{\partial\Omega}=0$. Here Ω is a bounded domain in R^m whose boundary $\partial\Omega$ is assumed to be sufficiently smooth.

We assume that $a=a(x)$ continuous in $\bar{\Omega}$. As I. M. Gel'fand [2] pointed out, these problems arise in the theory of thermal self-ignition of a chemically active mixture of gases in a vessel. He showed, in the special case where Ω is an m-dimensional ball of radius r, that for $m=1$ or 2 there exists a critical radius r_c such that BVP has two solutions, one solution or no solution, according as $0<r<r_c$, $r=r_c$ or $r>r_c$. If $m=3$, then the number of solutions of BVP can be 0, 1, 2, $\cdots$, or ∞, depending on r. It would be quite interesting to extend Gel'fand's result to the case of general Ω. However, we do not try to proceed in this direction. Instead, our main objective is to prove a certain relationship among solutions of BVP when they exist and to study the asymptotic stability of these solutions, i.e., the convergence of solutions of IVP to solutions of BVP as $t\to+\infty$. Below we describe some of our results. The other results and technical details will be published elsewhere together with generalizations including replacement of the function e^u by a general function $f=f(u)$; $R^1\to R^1$ which is smooth, positive, increasing, and strictly convex. The author wishes to thank Professor Melvyn Berger who brought the author's attention to the present problem and provided the author with helpful preliminary information.

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