CURVATURE STRUCTURES AND CONFORMAL TRANSFORMATIONS

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1. The notion of a "curvature structure" was introduced in §8, Chapter 1 of [1]. In this note we shall consider some of its applications. The details will be presented elsewhere.

Let (M, g) be a Riemann manifold. Whenever convenient, we shall denote the inner product defined by g, by $\langle\ \rangle$.

DEFINITION. A curvature structure on (M, g) is a $(1, 3)$ tensor field T such that, for any vector fields X, Y, Z, W on M,

$$T(X, Y) = -T(Y, X) \tag{1}$$

$$\langle T(X, Y)Z, W\rangle = \langle T(Z, W)X, Y\rangle \tag{2}$$

$$T(X, Y)Z + T(Y, Z)X + T(Z, X)Y = 0. \tag{3}$$

Such a curvature structure naturally defines the corresponding "sectional curvature" K_T which is a real valued function on $G_2(M)$, the Grassmann bundle of 2-planes on M; namely, for $x \in M$, $\sigma = \{X, Y\}$ a 2-plane at x,

$$K_T(\sigma) = \frac{\langle T(X, Y)X, Y\rangle}{\langle X, X\rangle\langle Y, Y\rangle - \langle X, Y\rangle^2}.$$

As the following results show, these sectional curvature functions are of considerable geometric interest.

2. Examples of curvature structures.

(a) *A trivial curvature structure.* Consider the $(1, 3)$ tensor field I given by

$$I(X, Y)Z = \langle X, Z\rangle Y - \langle Y, Z\rangle X.$$

In this case, $K_I \equiv$ constant.

(b) *Riemann curvature structure.* This is the usual curvature structure defined by the metric g; namely, if ∇ denotes the corresponding covariant derivative,

$$R(X, Y)Z = \nabla_{[X,Y]}Z - [\nabla_X, \nabla_Y]Z.$$

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