SPACES DETERMINED BY A GROUP OF FUNCTIONS

BY C. L. WIGINTON AND SUSAN SHRADER

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1. **Introduction.** Let G_F denote the group of all homeomorphisms of the topological space F onto itself, and let $G_{F'}$ be similarly defined for a space F'. If G_F and $G_{F'}$ are topologized under the point open topology, and if there is a function from G_F onto $G_{F'}$ which is a homeomorphism as well as an algebraic isomorphism then Wechsler [1] has determined a sufficient condition for the spaces F and F' to be homeomorphic. Thomas [2] has recently generalized Wechsler's theorem by weakening this condition on the spaces F and F'. It is the purpose here to generalize Wechsler's theorem in a different direction by using a group of functions other than a group of homeomorphisms.

2. **Preliminaries.** Most of our notation can be found in [1] and [2]; for reference we include the following. The space F is *n-homogeneous with respect to a group of functions* G provided for any pair of proper n-tuples $(x_1, \cdots, x_n)$, $(y_1, \cdots, y_n)$, there is a g in G such that $g(x_i) = y_i$, $i = 1, \cdots, n$. The space F is *ω-homogeneous with respect to a group of functions* G provided it is n-homogeneous with respect to G for each positive integer n.

Let $G_x = \{f \in G : f(x) = x\}$. Then G_x is a subgroup of G and will be called the *subgroup of the point* x. Furthermore G/G_x will denote the set of left cosets, and cosets will be written as fG_x.

We will use the point open topology on G and will consider G/G_x to have the topology induced by the natural mapping, that is, $\nu_x : G \to G/G_x$ defined by $\nu_x(h) = hG_x$ is to be continuous so that a set U is open in G/G_x if and only if $\nu_x^{-1}(U)$ is open in G. All spaces are T_2.

Our main theorem is as follows:

THEOREM 1. *Let F be a topological space, and let G denote a group of one-to-one functions from F onto itself with respect to which F is ω-homogeneous, and let F' and G' be similarly defined. Suppose that Φ is a homeomorphism from G onto G' such that Φ is an isomorphism. Then there is a homeomorphism from F onto F'.*

The proof of the main theorem will be accomplished by showing the existence of a sequence of homeomorphisms whose composition will then be the desired homeomorphism between F and F'. We prove first that G/G_x is homeomorphic to F. We then show that Φ induces a homeomorphism from G/G_x onto $G'/\Phi(G_x)$. It is next shown that the