A GALOIS PROBLEM FOR MAPPINGS

BY DAVID G. MOWAT[1]

Communicated by Gion-Carlo Rota, June 3, 1968

1. **Introduction.** A closure space (A, J) consists of a complete lattice A and a closure operator J defined on A. Given two closure spaces (A, J) and (B, K), and a supremum preserving mapping $f: A \to B$, we say that f is continuous if $f^{\Delta}(x)$ is J-closed in A whenever x is K-closed in B, where $f^{\Delta}: B \to A$ is the infimum preserving mapping given by

$$f^{\Delta}(x) = \sup\{z \in A \mid f(z) \leqq x\}.$$

If A is a complete lattice, (B, K) a closure space and $f: A \to B$ a supremum preserving mapping, then $f^{\Delta}Kf$ is the largest closure operator on A which makes f continuous. In fact, given any family X of supremum preserving mappings from A into (B, K) there exists a unique largest closure operator $\Gamma(X)$ on A which makes all the mappings in X continuous. Conversely, we may associate with each closure operator J on A the family $F(J)$ of all continuous supremum preserving mappings from (A, J) into (B, K). It is easily verified that the correspondences $[\Gamma, F]$ establish a Galois connexion between the set of all families of supremum preserving mappings from A into (B, K) and the set of all closure operators on A. We now wish to determine the Galois closed elements for this Galois connexion, that is, we wish to characterize those closure operators J on A and those families X of supremum preserving mappings from A into (B, K) for which $\Gamma F(J) = J$ and $F\Gamma(X) = X$.

2. **The main theorems.** For the Galois connexion described above, the fact that the set of all closure operators on A, ordered pointwise, is a co-atomistic complete lattice may be used to characterize the Galois closed closure operators on A.

THEOREM 1. *Let A be a complete lattice, and (B, K) a closure space. If K is not the indiscrete closure operator, then every closure operator on A is Galois closed. If K is the indiscrete closure on B, then the indiscrete closure is the only Galois closed closure operator on A.*

---

[1] This work comprised part of the author's dissertation written under Professor Henry H. Crapo at the University of Waterloo, 1968.