CONSTRUCTING 3-MANIFOLDS FROM GROUP HOMOMORPHISMS

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1. **Introduction.** Let S be a closed, orientable 2-manifold of genus $n>0$. Let F_1 and F_2 be free groups of rank n and denote by $F_1 \times F_2$ their direct product. Fix a point s_0 of S and suppose η_1, η_2 are homomorphisms of $\pi_1(S, s_0)$ *onto* F_1 and F_2 respectively. The homomorphism

$$\eta_1 \times \eta_2 \colon \pi_1(S, s_0) \to F_1 \times F_2$$

is called a *splitting homomorphism of* $\pi_1(S, s_0)$. Let M be a closed, orientable 3-manifold. In [**3**] J. Stallings introduced a natural splitting homomorphism induced by a Heegaard splitting of M. The purpose of this paper is to announce that for any splitting homomorphism there is a closed, orientable 3-manifold M and a Heegaard splitting of M so that the induced splitting homomorphism is equivalent to the given splitting homomorphism. This is Theorem 4.1 of §4. See §2 for definitions.

It is shown in Theorem 4.2 that two conjectures made by J. Stallings in [**3**] are true if and only if Poincaré's Conjecture that any closed, simply-connected 3-manifold is a 3-sphere, is true. These conjectures appear in §4 as Conjecture B and Conjecture D (using the notation of [**3**]).

Perhaps of independent interest is the Corollary to Lemma 3.2 of §3. It states that there is a homomorphism of the fundamental group of a closed, orientable surface of genus n *onto* a free group of rank k iff $k \leqq n$.

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2. **Notation and definitions.** The term *map* is used to mean continuous function. If f is a map from (S, s_0) to (X, x_0), then the homomorphism of $\pi_1(S, s_0)$ to $\pi_1(X, x_0)$ induced by f is denoted f_*. Suppose l is a map of S^1 into a pathwise connected space S. Then l

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