

## REFERENCES

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UNIVERSITY OF MINNESOTA

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## THE UNION OF FLAT $(n-1)$ -BALLS IS FLAT IN $R^n$

BY ROBION C. KIRBY<sup>1</sup>

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**THEOREM.**<sup>2</sup> *Let  $\beta_1^{n-1}$  and  $\beta_2^{n-1}$  be two locally flat  $(n-1)$ -balls in  $R^n$  with  $\beta_1 \cap \beta_2 = \partial\beta_1 \cap \partial\beta_2 = \beta^{n-2}$ , where  $\beta^{n-2}$  is an  $(n-2)$ -ball which is locally flat in  $\partial\beta_1$  and  $\partial\beta_2$ . Then  $\beta_1 \cup \beta_2$  is a flat  $(n-1)$ -ball in  $R^n$ .*

This result has been announced by Černavskii [1], but only for  $n \geq 5$  since his outlined proof uses engulfing. Our proof avoids engulfing and works for all  $n$ ; a thorough knowledge of Cantrell and Lacher's version (see [2, §§4 and 5]) of Černavskii's theorem is necessary to understand our proof.

We also have another proof of the following corollary which appears in [4].

**COROLLARY.** *Let  $g: M^{n-1} \rightarrow N^n$  be an imbedding of an  $(n-1)$ -manifold into an  $n$ -manifold which is locally flat except on a set  $E$ . If  $n > 3$ , then  $E$  contains no isolated points (see [3] for the same result when  $M$  and  $N$  are spheres).*

**PROOF.** Let  $C$  be a neighborhood of an isolated point  $p$  in  $M$  which is homeomorphic to an  $(n-1)$ -ball, with  $g$  locally flat on  $C-p$ . Then split  $C$  into  $(n-1)$ -balls  $C_1$  and  $C_2$  so that  $C = C_1 \cup C_2$  and  $C_1 \cap C_2$  is an  $(n-2)$ -ball containing  $p$ .  $g$  is locally flat on  $C_1$  and  $C_2$  except at the point  $p$  on their boundaries. Then, since  $n > 3$ ,  $g$  is flat on all of  $C_1$  and  $C_2$  by [5]. It follows from the theorem that  $C_1 \cup C_2 = C$  is flat, so  $E$  has no isolated points.

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<sup>2</sup> *Added in proof.* Černavskii has independently proven this theorem by similar methods.