SUMMABILITY VIEWED AS INTEGRATION[1]

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1. Let $s = \{s_n\}$ denote an infinite sequence of complex numbers and let $A = (a_{nk})$ be a summation matrix. If the A-transform of s, $\{t_n\} = \{ \sum_{k=0}^{\infty} a_{nk}s_k \}$ is a bounded sequence, it may be regarded as a bounded continuous function $t(n)$ on the discrete space of natural numbers N, and thus it has a continuous extension $\bar{t}$ to βN, the Stone-Čech compactification of N, cf. [2, pp. 82–95]. Let γ_0 be a fixed point of $\beta N - N$; we define

$$\int_N sdA = \bar{t}(\gamma_0)$$

to obtain a finitely additive integration process on N. In particular $\int_N sdA = \sigma$ whenever the matrix A evaluates s to σ.

Analogously an integration process on N can be created from summation methods arising from sequence to function transformations. For example if $\mathfrak{A}$ is the Abel method, we choose a point ρ_0 in $\beta I - I$, where I denotes the interval $[0, 1)$, and define, for all sequences $\{s_n\}$ such that $S(x) = (1-x) \sum_{n=0}^{\infty} s_n x^n$ converges for $|x| < 1$ and is bounded on I,

$$\int_N sd\mathfrak{A} = \tilde{S}(\rho_0),$$

where $\tilde{S}$ is the extension of S to I. The Abel method gives rise to a translation invariant integration on N.

In this note we shall study the function and in particular the Fourier analysis of the integration described. Each summation method will be identified with the measure or integration on N which it defines. All measures will be assumed to be regular summation methods on the set of null sequences; if the measure is representable by a matrix (a_{nk}) this means

$$\lim_{n\to\infty} a_{n,k} = 0, \qquad \text{lub} \sum_{k=0}^{\infty} | a_{nk} | < \infty. \tag{1}$$

REMARK. *The only countably additive summation methods ϕ are those of the form*

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