A BASIS FOR THE LAWS OF PSL(2,5)

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1. **Introduction.** Although it is known that there is a finite basis for the laws of any finite group (Sheila Oates and M. B. Powell [6]), it is not in general an easy matter to find an explicit basis for the laws of a given finite group. Indeed, the set of laws given below is, as far as we know, the only explicit basis known for the laws of a finite non-abelian simple group.

Before writing down the basis we define the law u_n introduced by L. G. Kovács and M. F. Newman [4]:

$$u_3 = \left[(x_1^{-1}x_2)^{x_{1,2}}, (x_1^{-1}x_3)^{x_{1,3}}, (x_2^{-1}x_3)^{x_{2,3}}\right]$$

and, for $n > 3$,

$$u_n = \left[u_{n-1}, (x_1^{-1}x_n)^{x_{1,n}}, \cdots, (x_{n-1}^{-1}x_n)^{x_{n-1,n}}\right].$$

THEOREM A. *The set of laws* (1)–(7) *given below is a basis for the laws of* PSL(2, 5), *the simple group of order* 60.

(1) $x^{30} = 1$

(2) $\{(x^{10}y^{10})^6[x^{10}, y^{10}]^2\}^5 = 1$

(3) $\{((x^6y^{12})^5(x^6y^{18})^5)^3[x^6, y^6]^6\}^6 = 1$

(4) $[x^3, y^3]^{15} = 1$

(5) $\{[x^6y^{10}x^{-6}, y^{-10}][y^{10}, x^6]\}^{10} = 1$

(6) $\{[y^{10}x^6y^{-10}, x^{-6}][y^{10}, x^6]^2\}^6 = 1$

(7) $u_{61} = 1.$

It can be verified by direct calculation that PSL(2, 5) satisfies these laws, so it is sufficient to prove that the variety $\mathfrak{V}$ defined by these laws is contained in the variety $\mathfrak{V}_0$ generated by PSL(2, 5).

2. **Notation.** In notation and terminology we will follow the book of Hanna Neumann [5]; we will also assume familiarity with the results of Chapters 1 and 5 of this book.

3. **Finite soluble groups in $\mathfrak{V}$.**

LEMMA 3.1. *Groups in $\mathfrak{V}$ of prime-power order are elementary abelian.*

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