COHOMOLOGY OF NONASSOCIATIVE ALGEBRAS

BY NEAL D. GLASSMAN

Communicated by G. D. Mostow, January 15, 1968

The theories of associative and Lie cohomology for finite dimensional algebras over fields have much in common. Let $\mathfrak{A}$ be an associative algebra with unit over the field K, M a unital $\mathfrak{A}$-bimodule; let $\mathfrak{L}$ be a Lie algebra over K, N an $\mathfrak{L}$-bimodule. Define $H^n(\mathfrak{A}, M) = \mathrm{Ext}^n_{\mathfrak{A}\otimes_K\mathfrak{A}^0}(\mathfrak{A}, M)$, $H^n(\mathfrak{L}, N) = \mathrm{Ext}^n_{U(L)}(K, N)$. Here $\mathfrak{A}^0$ is the opposite algebra of $\mathfrak{A}$, $U(\mathfrak{L})$ is the universal enveloping algebra of $\mathfrak{L}$, $\mathfrak{A}$ is regarded as the regular $\mathfrak{A}$-bimodule, and K is regarded as a trivial $\mathfrak{L}$-bimodule. We find that

(i) $H^1(\mathfrak{A}, M)$, $H^1(\mathfrak{L}, N)$ are naturally isomorphic to the K-vector-spaces of derivations from the algebra to the bimodule modulo the inner derivations from the algebra to the bimodule.

(ii) $H^0(\mathfrak{A}, M)$, $H^0(\mathfrak{L}, N)$ are naturally isomorphic to the sub-K-vector spaces of M, N respectively that determine the inner derivation 0—i.e. $H^0(\mathfrak{A}, M)$ is naturally isomorphic to the K-vector-space generated by $\{m \in M \mid m_R - m_L = 0\}$ and $H^0(\mathfrak{L}, N)$ is naturally isomorphic to the K-vector-space generated by $\{n \in N \mid n_R = 0\}$.

(iii) $H^2(\mathfrak{A}, M)$, $H^2(\mathfrak{L}, N)$ are naturally isomorphic to the K-vector-spaces of equivalence classes of short singular extensions of M by $\mathfrak{A}$, N by $\mathfrak{L}$, respectively.

(iv) $H^n(\mathfrak{A}, M)$, $H^n(\mathfrak{L}, N)$, $n \geqq 3$, are naturally isomorphic to the K-vector-spaces of equivalence classes of singular extensions of length n of M by $\mathfrak{A}$, N by $\mathfrak{L}$, respectively.

We construct a cohomology theory for an arbitrary nonassociative algebra satisfying a set of identities T, within which the associative and Lie theories are special cases. Let $\mathfrak{A}$ be a T-algebra over the commutative ring with unit K, M a T-bimodule for $\mathfrak{A}$. We write $U(\mathfrak{A})$ for the universal multiplication algebra of $\mathfrak{A}$; that is, $U(\mathfrak{A})$ is an associative algebra with unit such that all $\mathfrak{A}$-bimodules are right unital $U(\mathfrak{A})$-modules in a natural fashion and conversely. For details of this, see Jacobson [**4a**] or Knopfmacher [**5**].

Following Gerstenhaber, we make the next two definitions.

DEFINITION. $H^2(\mathfrak{A}, M)$ is the K-module of (not necessarily K-split) equivalence classes of short singular extensions of M by $\mathfrak{A}$.

DEFINITION. $H^n(\mathfrak{A}, M)$, $n \geqq 3$, is the K-module of (not necessarily K-split) equivalence classes of singular extensions of length n of M by $\mathfrak{A}$.