SOME AUTOMORPHISM GROUPS IN NOETHERIAN DOMAINS

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Suppose R is a Noetherian domain. Then a quasi-multiplication $f: R \to R$ is a function f such that $f(x) \in (x)$, for all $x \in R$. A function $f: R \to R$ such that $\partial f(a, x) = f(a+x) - f(a)$ is a quasi-multiplication, is q-differentiable at a. In [1] we studied quasi-multiplications on general R-modules, R commutative. In particular, we obtained some criteria for a submodule to be an analytic continuation submodule, i.e. a submodule such that the only everywhere differentiable extension of 0 on the submodule to the module is 0. It is shown (and clear) that an additive map is everywhere q-differentiable if and only if it is a quasi-multiplication.

In this paper we show that if R is a Noetherian domain, then the set of everywhere differentiable automorphisms is a normal subgroup of the group of automorphisms of R. With these automorphisms one can duplicate to a certain extent the constructions made with analytic automorphisms on complete discrete valuation rings. In particular, one can define analogues of the pseudo-ramification groups and the groups of strongly inertial automorphisms. We show that in a Noetherian domain every nonzero ideal is an analytic continuation ideal relative to the everywhere q-differentiable automorphisms, i.e. every automorphism of a nonzero ideal which is everywhere q-differentiable has a unique extension to an everywhere q-differentiable automorphism on the entire domain. As a consequence we see that if the ideal is an analytic continuation ideal, then these automorphisms can only be extended to automorphisms if we permit extensions out of the collection of everywhere q-differentiable maps on R.

The Groups G_R, $G_{\{\bar{a}\}}$, $G_{\langle\bar{a}\rangle}$, $G_{\bar{a}}$. If R is a Noetherian domain and if $\bar{a} \neq (0)$ is an ideal, then we define the following sets:

$$G_R = \{\sigma \mid \sigma \text{ is an automorphism and a quasi-multiplication}\},$$

$$G_{\{\bar{a}\}} = \{\sigma \in G_R \mid \sigma(x) - x \in \bar{a} \text{ for all } x \in R\},$$

$$G_{\langle\bar{a}\rangle} = \{\sigma \in G_R \mid \sigma(x) = u(x)x,\ u(x) \in 1 + \bar{a} \text{ for all } x \in R\}.$$

The set $G_{\bar{a}}$ is defined as G_R.

THEOREM 1. *If G is the group of automorphisms on R, then*

$$G_R \lhd G, \qquad G_{\{a\}} \lhd G_R, \qquad G_{\langle a\rangle} \lhd G_R.$$