INTERIORITY OF A HOLOMORPHIC MAPPING ON THE SET OF ITS EXCEPTIONAL POINTS[1]

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I. **Introduction.** A mapping $f: A \to B$ is said to be *interior* (or *open*) if for every open subset $U \subset A$, $f(U)$ is an open subset of B; it is said to be interior at a point $a \in A$ (or *locally interior* at $a \in A$) if for every open subset $U \subset A$ containing a, $f(a)$ is an interior point of $f(U)$. Clearly a mapping is interior if and only if it is locally interior everywhere on its domain of definition.

The result contained in this note is about the local interiority property of a holomorphic mapping on the set of its exceptional points. We shall restrict our attention to holomorphic mappings $f = (f_1(x), \cdots, f_n(x)): D \to C^n$ where D is a domain (open connected set) in $\overline{C}^n$. $\overline{C}^n = \overline{C}^1 \times \cdots \times \overline{C}^1$ where $\overline{C}^1$ is the extended plane of each one of the complex variables x_i. f is said to be holomorphic when each one of the functions f_i is holomorphic on D. Let $J(x)$ be the value of the Jacobian of f at $x \in D$.

The set E of exceptional points of f is by definition $E = \{a \in D \mid a$ is not an isolated point of $f^{-1}f(a)\}$.

II. **Result.** We recall that if $a \notin E$, f is interior at a. In fact, if $a \notin E$ and $J(a) \neq 0$, the property follows immediately from the inverse function theorem (f is a local homeomorphism); if $a \notin E$ and $J(a) = 0$, it follows from a theorem of Osgood [1] (f maps finitely-to-one sufficiently small neighborhoods of a onto neighborhoods of $b = f(a)$). Our result pertains to the case $a \in E$:

THEOREM. *Let $f: D \to C^n$, $D \subset \overline{C}^n$, be a holomorphic mapping and let E be the set of exceptional points of f, then the subset E_0 in E such that $E_0 = \{x \in E \mid f$ is interior at $x\}$ is either the empty set or a set of isolated points.*

PROOF. If E is empty, f is everywhere interior in D as shown above. If f is degenerate, i.e., $J(x) \equiv 0$, it is not difficult to show that $E = D$ and $E_0 = \{\varnothing\}$.

Let then f be not degenerate and E not empty. H. Cartan [2] proved that E is an analytic set and $E \subseteq W = \{x \in D \mid J(x) = 0\}$. Com-

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