NEW SIMPLE LIE ALGEBRAS OF TYPE D_4

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This brief note is to demonstrate the existence of a new class of (exceptional) Lie algebras of type D_4. The construction stems from a cyclic sixth degree extension P/Φ, together with an element γ of norm 1 in the unique cubic subfield F/Φ of P/Φ, where $\gamma \notin N_{P/F}(P^*)$. Each such γ will determine a non-Jordan (see [1] for definition) Lie algebra $\mathfrak{L}(\gamma)$, of type $D_{4\text{III}}$. Two algebras of this form, $\mathfrak{L}(\gamma)$ and $\mathfrak{L}(\rho)$, will be isomorphic if and only if γ differs from a conjugate of ρ by a norm in $N_{P/F}(P^*)$. The possibility of obtaining new D_4's from such a construction was first conjectured in [2].

We shall make free use of the well-known theory of finite Galois descent for nonassociative algebras and all the results which we use may be found in ([5, Chapter 10]).

0. Preliminaries. We assume without further mention that all fields which appear here have characteristic unequal to 2 or 3.

Let $\mathfrak{J}$ be a split exceptional central simple Jordan algebra over P, $\{e_1, e_2, e_3\}$ a set of supplementary orthogonal primitive idempotents and let $\mathfrak{D} = \mathfrak{D}(\mathfrak{J}/\Sigma Pe_i)$ be the subalgebra of the derivation algebra of $\mathfrak{J}$ annihilating ΣPe_i. Then $\mathfrak{D}$ is the split D_4. If $\mathfrak{L}$ is a Φ-algebra form of $\mathfrak{D}$ $(P \supset \Phi)$, then we let $\mathfrak{L}^*$ be the Φ-subalgebra of $\text{End}_P \mathfrak{J}$ generated by $\mathfrak{L}$ (we view $\mathfrak{L}$ as a Φ-subspace of $\mathfrak{D}$ which contains a Φ-basis which is also a P-basis for $\mathfrak{D}$). It is known that $(\mathfrak{L}^*)_P \cong P_8 \oplus P_8 \oplus P_8$. $\mathfrak{L}$ is special (i.e., has the form $\mathfrak{s}(\mathfrak{A}, J)$ where $(\mathfrak{A}, J)$ is a central simple associative algebra of degree 8 with involution) if and only if $\mathfrak{L}^*$ has proper ideals. When $\mathfrak{L}^*$ is simple, i.e. when $\mathfrak{L}$ is exceptional, then $\mathfrak{L}$ is of known type—a Jordan D_4—if and only if $\mathfrak{L}^*$ is a total matrix algebra over its center. $\mathfrak{L}$ is of type $D_{4\text{III}}$ ($D_{4\text{VI}}$) if the center of $\mathfrak{L}^*$ is a cyclic (noncyclic) extension of Φ.

If $\mathfrak{L}$ is of type $D_{4\text{III}}$ and F is the center of $\mathfrak{L}^*$—the canonical $D_{4\text{I}}$-field extension of $\mathfrak{L}$—then $\mathfrak{L}$ is a non-Jordan $D_{4\text{III}}$ if and only if none of the simple components of $(\mathfrak{L}^*)_F$ is a total matrix algebra.

We shall need some technical information about the structure of split Cayley algebra. For this we refer to [6] and for convenience list the results below for reference.

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