

2. ———, *An invariant formulation of the new maximum-minimum theory of eigenvalues*, J. Math. Mech. **16** (1966) 213–218.
3. S. Goldberg, *Unbounded linear operators: Theory and applications*, McGraw-Hill, New York, 1966.
4. I. C. Gohberg and M. G. Kreĭn, *Fundamental theorems on deficiency numbers, root numbers, and indices of linear operators*, Uspehi Mat. Nauk **12** (1957), 43–188; English transl., Amer. Math. Soc. Transl. (2) **13** (1960), 185–264.
5. W. Stenger, *The maximum-minimum principle for the eigenvalues of unbounded operators*, Notices Amer. Math. Soc. **13** (1966), 731.
6. ———, *On the variational principles for eigenvalues for a class of unbounded operators*, J. Math. Mech. **17** (1968), 641–648.

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ON AN ADDITIVE DECOMPOSITION OF FUNCTIONS OF SEVERAL COMPLEX VARIABLES

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Communicated by Maurice Heins, November 16, 1967

1. Introduction. Recent attempts (see [1] and the references in the same article) to extend the Wiener-Hopf technique for functions of a single complex variable to those of two or more complex variables have relied on a remark of Bochner's [2] that guarantees the required decomposition under suitable restrictions. Bochner's remark states that: *if $f(z_1, \dots, z_n)$, $z_j = x_j + iy_j$, is analytic in a tube $T: \gamma_i < x_i < \delta_i$, $y_i \in (-\infty, \infty)$, and if $\int_{-\infty}^{\infty} \dots \int |f(z_1, \dots, z_n)|^2 dy_1 \dots dy_n$ converges in T , then there exists in T a decomposition $f = \sum_{i=1}^{2^n} f_i$, where each f_i is analytic and bounded in an octant shaped tube T_i containing the interior of T . Moreover, such a decomposition is unique up to additive constants.* The uniqueness of the decomposition is not verified in [2] but reference is made to H. Bohr's [3] corresponding result for functions of a single complex variable.

It is here shown that the uniqueness statement is false. However, the adjunction of the additional hypothesis that the $f_i \rightarrow 0$ when any one of the $x_j \rightarrow \infty$, in the tubes T_i , restores the uniqueness of the decomposition and justifies the use of the result in [2].

2. A counter-example. In the decomposition $f = \sum_{i=1}^{2^n} f_i$, f_1 is analytic and bounded in the tube $T_1: x_i > \gamma_i$, $y_i \in (-\infty, \infty)$, $i = 1, 2, \dots, n$, and f_2 is analytic and bounded in the tube $T_2: x_1 < \delta_1$, $x_j > \gamma_j$,