WIENER-HOPF OPERATORS AND ABSOLUTELY CONTINUOUS SPECTRA. II

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1. This paper is a continuation of [4]. It may be recalled that if A is a self-adjoint operator on a Hilbert space $\mathfrak{H}$ with spectral resolution $A=\int\lambda dE_\lambda$, then the set of elements x in $\mathfrak{H}$ for which $\|E_\lambda x\|^2$ is an absolutely continuous function of λ is a subspace, $\mathfrak{H}_a(A)$, of $\mathfrak{H}$ (see, e.g., Halmos [1, p. 104]). The operator A is said to be absolutely continuous if $\mathfrak{H}_a(A)=\mathfrak{H}$. As in [4], both spaces $L^2(0, \infty)$ and $L^2(-\infty, \infty)$ will be considered, but the underlying Hilbert space for the integral operators T and A occurring below will be $\mathfrak{H}=L^2(0, \infty)$.

As in [4], let $k(t)$ on $-\infty<t<\infty$ satisfy

$$k \in L^1(-\infty, \infty) \cap L^2(-\infty, \infty) \quad \text{and} \quad k(-t) = \bar{k}(t), \tag{1}$$

and let $K(\lambda)$ denote the (real-valued) function

$$K(\lambda) = \int_{-\infty}^{\infty} k(t)e^{i\lambda t}dt, \qquad -\infty < \lambda < \infty. \tag{2}$$

If the (bounded) operator T on $\mathfrak{H}$ is defined by

$$(Tf)(t) = \int_0^t k(s-t)f(s)ds, \qquad 0 \leqq t < \infty, \tag{3}$$

then the self-adjoint operator $A=T+T^*=2\mathrm{Re}(T)$ is given by

$$(Af)(t) = \int_0^\infty k(s-t)f(s)ds. \tag{4}$$

There will be proved the following

THEOREM. *If $k(t)$ satisfies* (1) *and if $k(t)\neq 0$ (a.e.) on $-\infty<t<\infty$, then the self-adjoint operator A of* (4) *is absolutely continuous and its spectrum is the closed interval*

$$\mathrm{sp}(A) = [\inf K(\lambda), \sup K(\lambda)], \tag{5}$$

where $K(\lambda)$ is defined in (2).

In [4] the absolute continuity of A was established under the hypothesis that $K(\lambda)\neq 0$ a.e. According to the above Theorem how-

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