FLATTENING A SUBMANIFOLD IN CODIMENSIONS ONE AND TWO

BY J. C. CANTRELL[1] AND R. C. LACHER[2]

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Let M and N be manifolds with $M \subset \text{Int } N$, and assume that $M-X$ is locally flat in N, where X is some subset of M. We are interested in finding conditions (intrinsic, placement, dimensional, etc.) which, when placed on X, imply that M is locally flat in N. Extremely useful and satisfying answers are provided by Bryant and Seebeck in [2], assuming that $\dim N - \dim M \geqq 3$. We announce here a method for deducing local versions of Corollary 1.1 of [2] in codimensions one and two.

DEFINITIONS. If M is a manifold, a *collaring* of Bd M in M is an embedding λ of Bd $M \times [0, \infty)$ into M such that $\lambda(x, 0) = x$ for each x in Bd M. We use R^n to denote euclidean n-space, B^n the closed unit ball in R^n.

THEOREM. *For integers $0 \leqq k < m \leqq n$, let D be an m-cell in R^n and let E be a k-cell in* Bd D. *Assume that the following condition is satisfied*:

$$D - E \text{ is locally flat in } R^n, \quad \text{and} \quad E \text{ is locally flat in Bd } D.$$

Then $(R^n, D) \approx (R^n, B^m)$ if and only if $\lambda(E \times I)$ is locally flat in R^n for some collaring λ of Bd D *in D.*

The proof of this theorem is similar to the proof of Theorem 4.2 of [7]. Theorem 4.1 of [7] must be used more carefully to replace Corollary 3.2 of [7].

A detailed proof of the above theorem, together with applications and generalizations, will appear elsewhere. We present below the immediate implications of [2]. (Actually, in an earlier paper which is in press, Bryant and Seebeck prove a local form of Corollary 1.1 of [2] which is enough to yield the following applications.)

REMARK. There are no dimensional restrictions (other than $0 \leqq k < m \leqq n$) in the above Theorem.

APPLICATION 1. *Let D be an m-cell in R^n, and let E be a k-cell in* Bd D. *Assume that*

$$D - E \text{ and } E \text{ are locally flat in } R^n, \quad \text{and} \quad E \text{ is locally flat in Bd } D.$$

If $k \leqq n-4$ then $(R^n, D) \approx (R^n, B^m)$.

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