1. **Summary.** The purpose of this note is to prove a "theorem of the alternative" for any "contraction mapping" $T$ on a "generalized complete metric space" $X$. The conclusion of the theorem, speaking in general terms, asserts that: *either* all consecutive pairs of the sequence of successive approximations (starting from an element $x_0$ of $X$) are infinitely far apart, *or* the sequence of successive approximations, with initial element $x_0$, converges to a fixed point of $T$ (what particular fixed point depends, in general, on the initial element $x_0$). The present theorem contains as special cases both Banach’s [1] contraction mapping theorem for complete metric spaces, and Luxemburg’s [2] contraction mapping theorem for generalized metric spaces.

2. **A fixed point theorem.** Following Luxemburg [2, p. 541], the concept of a "generalized complete metric space" may be introduced as in this quotation:

"Let $X$ be an abstract (nonempty) set, the elements of which are denoted by $x$, $y$, $\cdots$ and assume that on the Cartesian product $X \times X$ a distance function $d(x, y) (0 \leq d(x, y) \leq \infty)$ is defined, satisfying the following conditions

(D1) $d(x, y) = 0$ if and only if $x = y$,

(D2) $d(x, y) = d(y, x)$ (symmetry),

(D3) $d(x, y) \leq d(x, z) + d(z, y)$ (triangle inequality),

(D4) every $d$-Cauchy sequence in $X$ is $d$-convergent, i.e. $\lim_{n,m \to \infty} d(x_n, x_m) = 0$ for a sequence $x_n \in X$ ($n = 1, 2, \cdots$) implies the existence of an element $x \in X$ with $\lim_{n \to \infty} d(x, x_n) = 0$, ($x$ is unique by (D1) and (D3)).

This concept differs from the usual concept of a complete metric space by the fact that not every two points in $X$ have necessarily a finite distance. One might call such a space a generalized complete metric space."