LOCAL FUNCTIONALS AND GENERALIZED RANDOM FIELDS[1]

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Communicated by Henry McKean, October 13, 1967

1. **Introduction.** Let $\mathcal{K}(R^n)$ be the Schwartz space of infinitely differentiable real functions on R^n, the Euclidean n-space, with compact supports, and let F: $\mathcal{K}(R^n)\to\tilde{R}$ be a map where $\tilde{R}$ is the class of real random variables on a (fixed) probability space. Then F is said to be a *generalized random field* (*process if* $n=1$) if it is linear and continuous. Here continuity means that if $\{f_m\}\subset\mathcal{K}(R^n)$ and $f_m\to 0$ in the topology of $\mathcal{K}(R^n)$, then $F(f_m)\to 0$ in probability. Such an F is said to have *independent values* if $F(f)$ and $F(g)$ are mutually independent, whenever f, g in $\mathcal{K}(R^n)$ have disjoint supports. Let $M(\cdot)$: $\mathcal{K}(R^n)\to$scalars, be a functional such that (i) $M(\cdot)$ is bounded on bounded sets of $\mathcal{K}(R^n)$; (ii) $M(\cdot)$ is continuous in that for any $\epsilon>0$, there is a neighborhood V of zero in $\mathcal{K}(R^n)$ such that $f-g\in V$ implies $|M(f)-M(g)|<\epsilon$; and (iii) $M(f+g)=M(f)+M(g)$ whenever f and g have disjoint supports. Such a functional $M(\cdot)$ is termed *local* by Gel'fand and Vilenkin [4, Chapter III, §4.1, footnote 2] where they raised the problem of characterizing $M(\cdot)$. Local functionals play a key role in the theory of generalized random fields with independent values. A special form of $M(\cdot)$ was used by Gel'fand in his study of such generalized random processes (cf. [3] and [4]). The purpose of this note is to state some results on the characterizations of local functionals and, as applications, to present generalizations of the Lévy-Khintchine representation formulas for characteristic functionals of generalized random fields with independent values. These results extend and complete the fundamental work of Gel'fand (cf. [3] and [4]) in many ways. The proofs and subsidiary results will be given elsewhere.

The concept of generalized random processes was independently introduced by Gel'fand [3] and Itô [5] who studied the generalized processes with independent values and the generalized stationary processes, respectively (see also [10]). The latter results were extended to certain nonstationary random fields in [7]. Another aspect of the theory was considered by Urbanik [9], for processes with independent values.

[1] This research was supported under the NSF grants GP-5921 and GP-7678.