CONJUGATIONS ON COMPLEX MANIFOLDS AND EQUIVARIANT HOMOTOPY OF MU[1]

BY PETER S. LANDWEBER

Communicated by Pierre Conner, November 1, 1967

1. **Introduction.** Let $\rho\colon \Omega_*^U \to \mathfrak{N}_*$ denote the natural homomorphism from the stably complex bordism ring into the unoriented bordism ring. Milnor showed in [8] that the image of ρ consists of all squares $([M]_2)^2$ in $\mathfrak{N}_*$. Since $\mathfrak{N}_*$ is a polynomial algebra over Z_2, an epimorphism $R\colon \Omega_{2n}^U \to \mathfrak{N}_n$ is defined by the condition that $R^2=\rho$. Milnor made use of the following result of Conner and Floyd [3, p. 64]: if τ is a conjugation on a closed almost complex $2n$-manifold M, then the fixed point set $F(M)$ is an n-manifold and $[M]_2=([F(M)]_2)^2$ in $\mathfrak{N}_{2n}$, i.e. $R([M])=[F(M)]_2$. Hence, if a conjugation is present we may regard R as "passage to the fixed point set." We shall develop a bordism theory in which such a "fixed point homomorphism" is a natural feature.

From the homotopy point of view, Ω_*^U coincides with the (stable) homotopy $\pi_*(MU)$ of the Milnor spectrum MU [7]. In fact, the Thom spaces $MU(n)$ carry involutions making it possible to define equivariant homotopy groups $\Omega_{p,q}^U=\pi_{p,q}(MU)$. The details follow.

Give C^m the involution $(z_1, \cdots, z_m)\mapsto(\bar{z}_1, \cdots, \bar{z}_m)$. Then the Grassmannian $G_n(C^m)$ of n-planes in C^m inherits an involution, as does the classifying space $BU(n)=G_n(C^\infty)$. Moreover, the universal complex n-plane bundle $E^n\to BU(n)$ inherits an involution which makes E^n a real vector bundle over the real space $BU(n)$ in the sense of Atiyah [1]. Thus $MU(n)=B(E^n)/S(E^n)$ is endowed with an involution fixing the base point. Notice that the corresponding fixed point sets are R^m, $G_n(R^m)$, $BO(n)$ and $MO(n)$.

Following Atiyah [1] let $B^{p,q}$ and $S^{p,q}$ denote the unit ball and unit sphere in a Euclidean space $R^{p,q}$ of dimension $p+q$ carrying an orthogonal involution with fixed point set R^q. If X is a space with involution and fixed base point $*$, let $\pi_{p,q}(X)$ denote the set of equivariant homotopy classes of maps $(B^{p,q}, S^{p,q})\to(X, *)$. For $q\geqq 2$, $\pi_{p,q}(X)$ is an abelian group.

There are equivariant suspension maps $i_n\colon MU(n)\wedge(B^{1,1}/S^{1,1})\to MU(n+1)$, and so homomorphisms

$$\pi_{p+k,q+k}(MU(k)) \to \pi_{p+k+1,q+k+1}(MU(k+1)).$$

[1] This research was supported in part by National Science Foundation Grant GP-6567.