A FATOU-TYPE THEOREM FOR HARMONIC FUNCTIONS ON SYMMETRIC SPACES[1]

BY S. HELGASON AND A. KORÂNYI

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1. **Introduction.** The result to be proved in this article is that if u is a bounded harmonic function on a symmetric space X and x_0 any point in X then u has a limit along almost every geodesic in X starting at x_0 (Theorem 2.3). In the case when X is the unit disk with the non-Euclidean metric this result reduces to the classical Fatou theorem (for radial limits). When specialized to this case our proof is quite different from the usual one; in fact it corresponds to transforming the Poisson integral of the unit disk to that of the upper half-plane and using only a homogeneity property of the Poisson kernel. The kernel itself never enters into the proof.

2. **Harmonic functions on symmetric spaces.** Let G be a semisimple connected Lie group with finite center, K a maximal compact subgroup of G and $\mathfrak{g}$ and $\mathfrak{k}$ their respective Lie algebras. Let B denote the Killing form of $\mathfrak{g}$ and $\mathfrak{p}$ the corresponding orthogonal complement of $\mathfrak{k}$ in $\mathfrak{g}$. Let Ad denote the adjoint representation of G. As usual we view $\mathfrak{p}$ as the tangent space to the symmetric space $X=G/K$ at the origin $o=\{K\}$ and accordingly give X the G-invariant Riemannian structure induced by the restriction of B to $\mathfrak{p}\times\mathfrak{p}$. Let Δ denote the corresponding Laplace-Beltrami operator.

Fix a maximal abelian subspace $\mathfrak{a}\subset\mathfrak{p}$ and let M denote the centralizer of $\mathfrak{a}$ in K. If λ is a linear function on $\mathfrak{a}$ and $\lambda\neq 0$ let $\mathfrak{g}_\lambda=\{X\in\mathfrak{g}\mid [H,X]=\lambda(H)X \text{ for all } H\in\mathfrak{a}\}$; λ is called a restricted root if $\mathfrak{g}_\lambda\neq 0$. Let $\mathfrak{a}'$ denote the open subset of $\mathfrak{a}$ where all restricted roots are $\neq 0$. Fix a Weyl chamber $\mathfrak{a}^+$ in $\mathfrak{a}$, i.e. a connected component of $\mathfrak{a}'$. A restricted root α is called positive (denoted $\alpha>0$) if its values on $\mathfrak{a}^+$ are positive. Let the linear function ρ on $\mathfrak{a}$ be determined by $2\rho=\sum_{\alpha>0}(\dim\mathfrak{g}_\alpha)\alpha$ and denote the subalgebras $\sum_{\alpha>0}\mathfrak{g}_\alpha$ and $\sum_{\alpha>0}\mathfrak{g}_{-\alpha}$ of $\mathfrak{g}$ by $\mathfrak{n}$ and $\bar{\mathfrak{n}}$ respectively. Let N and $\overline{N}$ denote the corresponding analytic subgroups of G.

By a Weyl chamber in $\mathfrak{p}$ we understand a Weyl chamber in some maximal abelian subspace of $\mathfrak{p}$. The *boundary* of X is defined as the set B of all Weyl chambers in the tangent space $\mathfrak{p}$ to X at o; since this boundary is via the map $kM\to\mathrm{Ad}(k)\mathfrak{a}^+$ identified with K/M, which by the Iwasawa decomposition $G=KAN$ equals G/MAN, this defi-

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