CONSTRUCTIONS ON LOW-DIMENSIONAL DIFFERENTIABLE MANIFOLDS

BY VALENTIN POÉNARU[1]

Communicated by William Browder, May 29, 1967

1. This note contains the statements of three theorems on low-dimensional differentiable manifolds (dimensions 3 and 4). The proofs, which use techniques partly connected to [1], will appear elsewhere.

We denote by $M_n+(\phi_\lambda)$ the manifold obtained by adding the handle of index λ, $D_\lambda \times D_{n-\lambda}$, via the embedding ϕ_λ: $S_{\lambda-1}\times D_{n-\lambda} \to \partial M_n$, to M_n. More generally, we shall use the following notation: If $P_{n-1}\subset S_{n-1}=\partial D_n$ is a bounded submanifold and ψ: $P_{n-1}\to M_n$ an embedding, we denote by $M_n+(\psi)$, the space $M_n \cup D_n$, where every $x \in P_{n-1}$ is identified to $\psi(x)\in M_n$. It is understood that, if $\psi(P_{n-1}) \subset \partial M_n$, then $M_n+(\psi)$ is a "usual" differentiable manifold, otherwise a "singular" one (see §2).

THEOREM 1. *Let M_3 be a compact, differentiable, homotopy 3-disk. Then $M_3\times I$ is diffeomorphic to D_4 with handles of index 2 and 3 added*:

$$M_3 \times I = D_4 + (\overset{1}{\phi_2}) + \cdots + (\overset{p}{\phi_2}) + (\overset{1}{\phi_3}) + \cdots + (\overset{p}{\phi_3}).$$

Hence, one can eliminate the handles of index 1 of $M_3\times I$ (compare with the similar procedure, in higher dimensions [2]).

In fact we obtain Theorem 1 from the slightly stronger:

THEOREM 1′. *If M_3 is a compact, differentiable homotopy 3-disk, there exists an integer $p=p(M_3)$ such that*:

$$(M_3 \# (S_2 \times I) \# \cdots \# (S_2 \times I)(p \text{ times})) \times I = D_4 + (\overset{1}{\phi_2}) + \cdots + (\overset{p}{\phi_2}).$$

This together with some immersion theory, implies easily the main result from [1].

The next theorem is the main step in proving Theorem 1′. But in order to state it, we need some preparation.

2. We consider the 3-manifold $T_p=(S_1\times D_2) \# (S_1\times D_2) \# \cdots \# (S_1\times D_2)$ (p times) and its double $2T_p=(S_1\times S_2) \# (S_1\times S_2) \# \cdots \# (S_1\times S_2)$ (p times). (# means connected sum.) A family of p 2-by-2 disjoint embeddings ϕ_i: $S_1\to T_p$ $(i=1, \cdots, p)$ is called

[1] This research has been supported by NSF Grant GP-6299.