

A MATHEMATICAL THEORY OF OPTIMAL ECONOMIC DEVELOPMENT¹

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1. Introduction. The general aim of this article is two-fold; first, to show by means of examples the kinds of mathematical problems which arise from economic situations, and second, to illustrate the principal methods by which these problems can be attacked. I have endeavored to present this material in a manner appropriate both for the general mathematician and the mathematically oriented economist. For the former whose interest is probably mainly one of curiosity, I have tried to convey the general flavor of the subject—a sort of answer to the question “what is mathematical economics?” For the latter I hope that these examples will provide useful illustrations of the power of modern optimization theory in attacking economic problems.

For achieving the above purposes the subject of economic development seems a particularly suitable vehicle. The problems are easily described and can be formulated mathematically in a variety of ways. An economy consists of various “goods” which can be used for two purposes: they can either be “invested” in which case they produce more of themselves, or they can be “consumed” in which case they provide satisfaction or “utility.” The entire structure of the economy is given by specifying how these two operations can be carried on. The specification of how goods can be transformed into each other is called the *technology* of the model and the specification of how goods are transformed to satisfaction is called the *utility function*. Given this structure and some initial bundle of goods, the problem of optimal development is to decide at each point of time how much to invest and how much to consume in order to maximize utility summed over time in some suitable way.

Having arrived at a definition of an optimal development program one proceeds to ask the usual questions. Are there any such programs?

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