FIXED POINTS OF MAPS OF βN

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In this note all spaces are assumed to be uniformizable, and if P is a space then βP denotes a Stone-Čech compactification of P. If P is a discrete space we shall think of the points of $P^* = \beta P - P$ as the free ultrafilters on P. The letter N denotes the discrete space of natural numbers. The main results, Theorems A and C, read as follows:

THEOREM A. *If M is a discrete space, f is a homeomorphism of βM into itself, and if P_k is the set of all k-periodic points of f, then*

$$P_k = \mathrm{cl}(P_k \cap M).$$

In particular

THEOREM B. *If M is discrete, then no homeomorphism of βM into M^* has a fixed point.*

THEOREM C. *If M is a discrete space, and if x is a fixed point of a continuous mapping f of βM into itself, then each neighborhood of x contains an f-invariant neighborhood V of x (that means, with $f[V] \subset V$). In addition, V can be chosen of the form* $\mathrm{cl}\, X$, $X \subset M$.

REMARK 1. For $M = N$ Theorem B reads: no type is a relative type of itself, i.e. $\langle t, t \rangle \in \Phi$ for no type t (for terminology see [1]). Using the continuum hypothesis W. Rudin proved in [5] that N^* is not homogeneous. In [1] the present author proved the nonhomogeneity of N^* without using the continuum hypothesis. Theorem B gives another, very simple, proof: if h is a homeomorphism of N^* onto itself and $hx = y$, then evidently the sets of all relative types of x and y coincide. Thence, by Theorem B, if the type of a point x is a relative type of y, then $hx = y$ for no homeomorphism h of N^*. Note that this proof establishes the existence of exp exp $\aleph_0$ of orbits of the homeomorphism group of N^*.

REMARK 2. Theorem B in the particular case $M = N$ was announced in [2]. M. Katětov reproved that result in [3] (his proof works for any M with $f[M]$ an r-set, see Example 2 below). The original proof, which works in the countable case only, is also given below because it is of some interest in itself.

REMARK 3. It would be interesting to find an analogue of Theorem B for any continuous mapping of βM. The following examples suggest that the entire orbit of M must be considered.