SOME HOMOTOPY OF STUNTED COMPLEX PROJECTIVE SPACE

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1. **Introduction.** The 2-components of the stable homotopy groups $\pi^s_{2n+i}(CP/CP^{n-1})$ of stunted complex projective space are here tabulated, up to group extension, for $8 \leqq i \leqq 13$. For earlier work, including computation of these groups for $i \leqq 7$, see [8], [2], [7], [3], [4], and [5] as corrected by [6]. See [1] for odd components.

A result of Toda [8] relates these stable groups to the metastable homotopy groups of unitary groups as follows: Let $0 \leqq t < n$. Then $\pi^s_{2n+2t+1}(CP/CP^{n-1}) = \pi_{2n+2t+1}U(n)$, while there exists a commutative diagram with an exact row

$$\begin{array}{ccccccccc}
0 & \longrightarrow & Z & \longrightarrow & \pi^s_{2n+2t}(CP/CP^{n-1}) & \longrightarrow & \pi_{2n+2t}U(n) & \longrightarrow & 0 \\
 & & \Big\downarrow{\scriptstyle (n+t)!} & & \Big\downarrow{\scriptstyle h} & & & & \\
 & & Z & = & H_{2n+2t}(CP/CP^{n-1}) & & & &
\end{array}$$

in which h is the Hurewicz homomorphism.

In view of Toda's formula the value of h is needed to deduce $\pi^{-}_{2n+2t}U(n)$. We include this data as (2.3) and give in (2.5) the order of the image of each element of the 2-component of $\pi^s_{2n+i}S^{2n}$ in $\pi^s_{2n+i}(CP/CP^{n-1})$.

Our basic method is the stable homotopy exact couple resulting from the standard cell filtration of CP/CP^{n-1}. By naturality, differentials in the resulting spectral sequence for CP/CP^{n-1} may be computed in the analogous spectral sequence for CP. The study in [6] of this sequence for CP is the basis of the calculation here; a more detailed description of the calculation will appear elsewhere.

2. **Results on homotopy groups.**

THEOREM 2.1. *The 2-component of the torsion of the stable homotopy group* $\pi^s_{2n+i}(CP/CP^{n-1})$, $8 \leqq i \leqq 13$, *is given by Table* 2.2.

In (2.2) nZ_2 denotes the direct sum of n copies of Z_2, while A ? B denotes a group satisfying an exact sequence $0 \to A \to A$? $B \to B \to 0$. Note that $Z_2 + Z_2$? $2Z_2$ denotes $Z_2 + A$, where $A = Z_2$? $2Z_2$, rather