ASYMPTOTIC BEHAVIOR OF MEROMORPHIC FUNCTIONS WITH EXTREMAL DEFICIENCIES

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Let $f(z)$ be a meromorphic function; it is assumed that the reader is familiar with the following symbols of frequent use in Nevanlinna's theory

$$n(r,f),\quad N(r,f),\quad T(r,f),\quad \delta(\tau,f).$$

The lower order μ and the order λ of $f(z)$ are defined by the familiar relations

$$\liminf_{r\to\infty}\frac{\log T(r,f)}{\log r}=\mu,\qquad \limsup_{r\to\infty}\frac{\log T(r,f)}{\log r}=\lambda.$$

In addition to these classical concepts, we consider the *total deficiency* $\Delta(f)$ of the function f

$$\Delta(f)=\sum_{\tau}\delta(\tau,f)$$

where the summation is to be extended to all the values τ, finite or ∞, such that

(1) $$\delta(\tau,f)>0.$$

The number of deficient values of f, that is the number of distinct values of τ for which (1) holds, will be denoted by $\nu(f)$ ($\leqq +\infty$).

The investigation presented here leads to the proof of

THEOREM A. *Let $f(z)$ be a meromorphic function of lower order μ:*

(2) $$\tfrac{1}{2}<\mu<1,$$

and let the poles of $f(z)$ have maximum deficiency ($\delta(\infty,f)=1$). Then

(3) $$\Delta(f)\leqq 2-\sin\pi\mu.$$

Moreover, if equality holds in (3), *then*

(4) $$\nu(f)=2.$$

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