IMMERSIONS OF G-MANIFOLDS, G FINITE

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G denotes a finite group. If G acts on X, and $H \subset G$, $X_H = \{x;\ hx = x,\ h \in H\}$.

1. P.L. **G-manifolds.** A *G-polyhedron* is a polyhedron K together with a P.L. action of G on K: in particular a *P.L. G-manifold* is a G-polyhedron whose polyhedron is a manifold. *Maps, subspaces* of G-polyhedra are G invariant maps, subspaces of the underlying polyhedra. A *Euclidean G space* is the P.L. G-manifold underlying a finite dimensional complex representation of G. A *G ball (pair)* is an invariant ball (pair) in some Euclidean G space. A P.L. G-manifold is *locally-Euclidean (l.e.)* if it has a covering by open sets each isomorphic to an open set in a G ball. A pair (N, M), N a G-manifold and M an unbounded submanifold contained in int N, is *locally Euclidean* if at each point p of M it is like a stabilizer p ball pair.

The *regular neighbourhood theorem* [**4**], [**9**] holds for l.e. G-manifolds but not in general. For example let S be a Whitehead sphere [**8**] and B the star of a fixed vertex: CS the cone on S, collapses to CB, but the two are distinct G-manifolds.

If P is a G-polyhedron and K a triangulation of P in which G acts by vertex permutation, a *G block bundle* over P will mean a block bundle ξ over K (see [**5**]) and an action of G on ξ as a group of bundle automorphisms compatible with the inclusion of K in the total space $E(\xi)$ such that for each simplex δ of K and block β above δ, $(\beta, \delta) \approx (B \times \delta, \delta)$ as H spaces, for some H ball B, where $H =$ stabilizer δ. $E(\xi)$ is naturally a G polyhedron. If P is a l.e. unbounded G-manifold $(E(\xi), P)$ is a l.e. pair and conversely

THEOREM 1. *Let $(N^n, M^m_n$be a l.e. unbounded G-manifold and unbounded submanifold and suppose M is compact. $\exists n-m$ G block bundle ξ over M unique up to isomorphism and an embedding f: $E(\xi) \rightarrow N$ extending the inclusion of M. If g: $E(\xi) \rightarrow N$ is another such $\exists$ isotopy F_t of N mod M and an automorphism α of ξ with $g = F_1 \cdot f \cdot E(\alpha)$.*

2. P.L. **G-embeddings.** M and N will denote P.L. G-manifolds, M compact and both without boundary.

$E_G(M, N)$, $I_G(M, N)$, Homeo$_G(N)$ are the *semisimplicial complexes of embeddings of M in N, immersions of M in N, homeomorphisms of N.* A k simplex of Homeo$_G(N)$ is a G-homeomorphism of $\Delta^k \times N$ com-